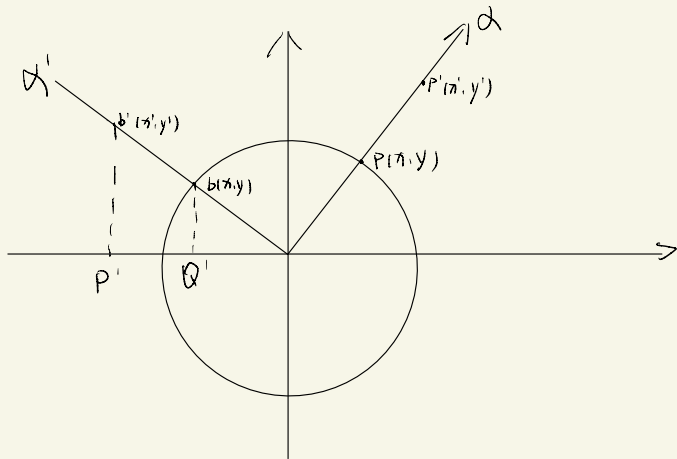






一、定义

$$1. \quad x = \cos \alpha \quad y = \sin \alpha \quad \frac{y}{x} = \tan \alpha \quad (\alpha \neq 0)$$

在单位圆上的交点.

$$\frac{x'}{r} = \cos \alpha \quad \frac{y'}{r} = \sin \alpha \quad \frac{y'}{x'} = \tan \alpha \quad r \text{ 为 } x, y \text{ 与原点的距离}$$

2. $\sin \alpha$ 在一、二象限上为正, 三、四象限上为负

$\cos \alpha$ 在一、四象限上为正, 二、三为负

$\tan \alpha$ 在一、三上为正, 二、四上为负

$$3. \quad \sin(\alpha + 2k\pi) = \sin \alpha \quad k \in \mathbb{Z}$$

$$\cos(\alpha + 2k\pi) = \cos \alpha$$

$$\tan(\alpha + 2k\pi) = \tan \alpha$$

$2\pi = 360^\circ$ (周角)

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0
$\tan \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	不存在	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	不存在

【例 3】(多选) 若 $\sin\theta\cos\theta < 0$, 则 θ 可能是 ()

(A) 第一象限角 (B) 第二象限角 (C) 第三象限角 (D) 第四象限角

【例 4】求下列各式的值: (1) $\sin 1140^\circ$; (2) $\cos(-\frac{4\pi}{3})$; (3) $\tan(-\frac{11\pi}{6})$.

$$\begin{cases} \sin(2k\pi + \alpha) = \sin \alpha \\ \cos(2k\pi + \alpha) = \cos \alpha \\ \tan(2k\pi + \alpha) = \tan \alpha \end{cases}$$

解: (1) $\sin 1140^\circ = \sin(1140^\circ - 160^\circ \times 3) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$$(2) \cos(-\frac{4\pi}{3}) = \cos(-\frac{4\pi}{3} + 2\pi) = \cos \frac{2\pi}{3} = -\frac{1}{2}$$

$$(3) \tan(-\frac{11\pi}{6}) = \tan(-\frac{11\pi}{6} + 2\pi) = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}.$$

【例 5】已知角 α 的终边上一点 $P(-2, 1)$, 则 $\sin\alpha =$ ____, $\tan\alpha =$ _____.

$$r = |OP| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$$

$$\sin\alpha = \frac{y}{r} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan\alpha = \frac{y}{x} = \frac{1}{-2} = -\frac{1}{2}.$$

$$\begin{cases} \cos\alpha = \frac{x}{r} \\ \sin\alpha = \frac{y}{r} \\ \tan\alpha = \frac{y}{x} \end{cases}$$

【例 6】已知角 α 的终边在直线 $y = -2x$ 上, 则 $\sin \alpha = \pm \frac{2}{\sqrt{5}}$.

$$\textcircled{1} x = 1, y = -2, r = \sqrt{5}, \sin \alpha = -\frac{2}{\sqrt{5}}$$

$$\textcircled{2} x = -1, y = 2, r = \sqrt{5}, \sin \alpha = \frac{2}{\sqrt{5}}$$

【例 7】已知角 α 的终边经过点 $P(m, -6)$, 且 $\cos \alpha = -\frac{4}{5}$, 则 $m =$ (B)

(A) 8 (B) -8 (C) 4 (D) -4

$$r = OP = \sqrt{m^2 + 36}$$

$$\cos \alpha = \frac{m}{r} = \frac{m}{\sqrt{m^2 + 36}}$$

$$\frac{m}{\sqrt{m^2 + 36}} = -\frac{4}{5}$$

$$-5m = 4\sqrt{m^2 + 36}$$

$$25m^2 = 16(m^2 + 36)$$

$$9m^2 = 16 \times 36$$

$$m^2 = 16 \times 4$$

$$m^2 = 64$$

$$m = \pm 8$$

$$\because m < 0$$

$$\therefore m = -8$$

【例 8】已知角 α 终边上一点 $P(a, -1) (a \neq 0)$, $\tan \alpha = -a$, 则 $\sin \alpha =$ (B)

(A) $\pm \frac{\sqrt{2}}{2}$ (B) $-\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $-\frac{1}{2}$

$$\tan \alpha = \frac{-1}{a} = -\frac{1}{a}, \quad r = \sqrt{a^2 + 1}$$

$$-\frac{1}{a} = -a$$

$$a = \pm 1$$

$$\sin \alpha = \frac{-1}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

