1. $\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$ $\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

2. $\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}$

3. $\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ $\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ $\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$

【例 1】已知 $\tan\frac{\alpha}{2}=2$，则 $\sin 2\alpha=$______.

$$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}=-\frac{4}{3}$$

$$\sin 2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}=-\frac{8}{3}\times\frac{9}{25}=-\frac{24}{25}$$

【例 2】已知$\sin\alpha=-\dfrac{4}{5}$，则$\tan\dfrac{\alpha}{2}=$ $\underline{-\frac{1}{2}\text{或}-2}$.

$$\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=-\frac{4}{5}$$

$$-10\tan\frac{\alpha}{2}=4+4\tan^2\frac{\alpha}{2}$$

$$2\tan^2\frac{\alpha}{2}+5\tan\frac{\alpha}{2}+2=0$$

$$(2\tan\frac{\alpha}{2}+1)(\tan\frac{\alpha}{2}+2)=0$$

$$\tan\frac{\alpha}{2}=-\frac{1}{2}\text{或}-2$$

【例 3】若$\dfrac{\sin\alpha}{1+\cos\alpha}=\dfrac{1}{2}$，则$\sin\alpha+\cos\alpha=$ $\underline{\frac{7}{5}}$.

$$\tan\frac{\alpha}{2}=\frac{1}{2}$$

$$\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{1}{\frac{5}{4}}=\frac{4}{5}$$

$$\cos=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{3}{5}$$

（1） $4\sin(60^\circ-\alpha)\sin\alpha\sin(60^\circ+\alpha)$；

（2） $\dfrac{\sin\alpha+\sin3\alpha+\sin5\alpha+\sin7\alpha}{\cos\alpha+\cos3\alpha+\cos5\alpha+\cos7\alpha}$；

（3） $\dfrac{4\cos^2\alpha}{\dfrac{1}{\tan\dfrac{\alpha}{2}}-\tan\dfrac{\alpha}{2}}$；

（4） $\dfrac{\sin4x}{1+\cos4x}\cdot\dfrac{\cos2x}{1+\cos2x}\cdot\dfrac{\cos x}{1+\cos x}$.

(1) 原式 $=4\sin\alpha\left(\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}\sin\alpha\right)\left(\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha\right)$

$=4\sin\alpha\left(\frac{3}{4}\cos^2\alpha-\frac{1}{4}\sin^2\alpha\right)$

$=4\sin\alpha\left(\frac{3}{4}\cdot\frac{1+\cos2\alpha}{2}-\frac{1}{4}\cdot\frac{1-\cos2\alpha}{2}\right)$

$=\sin\alpha\cdot\left(\frac{3+3\cos2\alpha}{2}-\frac{1-\cos2\alpha}{2}\right)$

$=\sin\alpha\cdot(1+2\cos2\alpha)$

$=\sin\alpha+\sin\alpha\cdot2\cos2\alpha$

$=\sin(2\alpha-\alpha)+\sin\alpha\cdot2\cos2\alpha$

$=\sin2\alpha\cdot\cos\alpha-\cos2\alpha\cdot\sin\alpha+2\cos2\alpha\cdot\sin\alpha$

$=\sin2\alpha\cdot\cos\alpha+\cos2\alpha\cdot\sin\alpha$

$=\sin3\alpha$

（1）$4\sin(60^\circ-\alpha)\sin\alpha\sin(60^\circ+\alpha)$；

（2）$\dfrac{\sin\alpha+\sin3\alpha+\sin5\alpha+\sin7\alpha}{\cos\alpha+\cos3\alpha+\cos5\alpha+\cos7\alpha}$；

（3）$\dfrac{4\cos^2\alpha}{\dfrac{1}{\tan\dfrac{\alpha}{2}}-\tan\dfrac{\alpha}{2}}$；

（4）$\dfrac{\sin4x}{1+\cos4x}\cdot\dfrac{\cos2x}{1+\cos2x}\cdot\dfrac{\cos x}{1+\cos x}$.

(2) 原式 $=\dfrac{\sin(4\alpha-3\alpha)+\sin(4\alpha+3\alpha)+\sin(4\alpha-\alpha)+\sin(4\alpha+\alpha)}{\cos(4\alpha-3\alpha)+\cos(4\alpha+3\alpha)+\cos(4\alpha-\alpha)+\cos(4\alpha+\alpha)}$

$=\dfrac{2\sin4\alpha\cos3\alpha+2\sin4\alpha\cos\alpha}{2\cos4\alpha\cos3\alpha+2\cos4\alpha\cos\alpha}$

$=\dfrac{\sin4\alpha\cos3\alpha+\sin4\alpha\cos\alpha}{\cos4\alpha\cos3\alpha+\cos4\alpha\cos\alpha}$

$=\dfrac{\sin4\alpha(\cos3\alpha+\cos\alpha)}{\cos4\alpha(\cos3\alpha+\cos\alpha)}$

$=\tan4\alpha$

(3) 原式 $= \dfrac{4\cos^2\alpha}{\dfrac{1}{\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}} - \sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}}$

$$= \frac{4\cos^2\alpha}{\dfrac{1-\dfrac{1-\cos\alpha}{1+\cos\alpha}}{\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}}}$$

$$= \frac{4\cos^2\alpha}{\dfrac{\dfrac{2\cos\alpha}{1+\cos\alpha}}{\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}}} \longrightarrow \frac{2\cos\alpha\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}}{1-\cos\alpha}$$

$$= \overset{2\cos\alpha}{\cancel{4\cos^2\alpha}} \cdot \frac{1-\cos\alpha}{\cancel{2\cos\alpha}} \quad \tan\frac{\alpha}{2}$$

$$= 2\cos\alpha \cdot \overset{\sqrt{1-\cos\alpha}}{\cancel{(1-\cos\alpha)}} \cdot \frac{\sqrt{1+\cos\alpha}}{\cancel{\sqrt{1-\cos\alpha}}}$$

$$= 2\cos\alpha \cdot \sin\alpha$$

$$= \sin 2\alpha$$

(4) 原式 $= \dfrac{\sin 4x}{2\cos^2 2x} \cdot \dfrac{\cancel{\cos 2x}}{2\cos^2 x} \cdot \dfrac{\cancel{\cos x}}{2\cos^2 \frac{x}{2}}$

$= \dfrac{\sin 4x}{8\cos 2x \cos x \cos^2 \frac{x}{2}}$

$= \dfrac{2\sin 2x \cos 2x}{8\cos 2x \cos x \cdot \frac{1+\cos x}{2}}$

$= \dfrac{\sin 2x}{2\cos x \cdot (1+\cos x)}$

$= \dfrac{2\sin x \cos x}{2\cos x (1+\cos x)}$

$= \dfrac{\sin x}{1+\cos x}$

$= \tan\frac{x}{2}$

（1）$4\sin(60^\circ-\alpha)\sin\alpha\sin(60^\circ+\alpha)$；

（2）$\dfrac{\sin\alpha+\sin 3\alpha+\sin 5\alpha+\sin 7\alpha}{\cos\alpha+\cos 3\alpha+\cos 5\alpha+\cos 7\alpha}$；

（3）$\dfrac{4\cos^2\alpha}{\dfrac{1}{\tan\frac{\alpha}{2}}-\tan\frac{\alpha}{2}}$；

（4）$\dfrac{\sin 4x}{1+\cos 4x}\cdot\dfrac{\cos 2x}{1+\cos 2x}\cdot\dfrac{\cos x}{1+\cos x}$.

【例 5】$\dfrac{\sqrt{3}\tan 12^\circ-3}{\sin 12^\circ(4\cos^2 12^\circ-2)}=$______.

原式 $= \dfrac{\sqrt{3}\tan 12^\circ - 3}{\sin 12^\circ(4\cos^2 12^\circ - 2\cos^2 12^\circ - 2\sin^2 12^\circ)}$

$= \dfrac{\sqrt{3}\tan 12^\circ - 3}{2\sin 12^\circ \cos 24^\circ}$

分子 $= \sqrt{3}\cdot\dfrac{\sin 12^\circ}{\cos 12^\circ} - 3 = \dfrac{\sqrt{3}\sin 12^\circ - 3\cos 12^\circ}{\cos 12^\circ} = \dfrac{2\sqrt{3}\sin(12^\circ - 60^\circ)}{\cos 12^\circ} = -\dfrac{2\sqrt{3}\sin 48^\circ}{\cos 12^\circ}$

分子 $= \sqrt{3}\cdot\dfrac{\sin 12^\circ}{\cos 12^\circ} - 3 = \dfrac{\sqrt{3}\sin 12^\circ - 3\cos 12^\circ}{\cos 12^\circ} = \dfrac{2\sqrt{3}\sin(12^\circ - 60^\circ)}{\cos 12^\circ} = -\dfrac{2\sqrt{3}\sin 48^\circ}{\cos 12^\circ}$

分母 $= 2\sin 12^\circ \cos 24^\circ$

原式 $= \dfrac{-\frac{2\sqrt{3}\sin 48^\circ}{\cos 12^\circ}}{2\sin 12^\circ \cos 24^\circ} = -\sqrt{3}\cdot\dfrac{\sin 48^\circ}{\sin 12^\circ \cos 12^\circ \cos 24^\circ} = -\sqrt{3}\cdot\dfrac{2\sin 48^\circ}{\sin 24^\circ \cos 24^\circ}$

$= -\sqrt{3}\cdot\dfrac{4\sin 48^\circ}{\sin 48^\circ} = -4\sqrt{3}$.