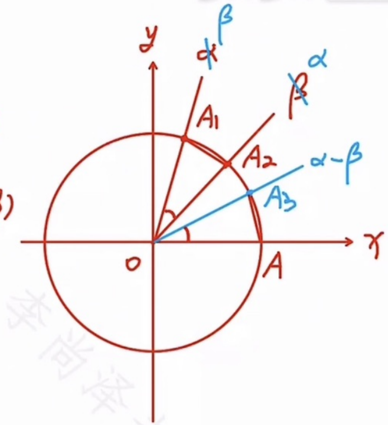
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$



$A_1(\cos\alpha,\ \sin\alpha)$ $\quad A_2(\cos\beta,\ \sin\beta)$ $\quad A_3(\cos(\alpha-\beta),\ \sin(\alpha-\beta))$ $\quad A(1,0)$

$|A_1A_2|=|AA_3|$ $\quad \therefore (\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2=[\cos(\alpha-\beta)-1]^2+\sin^2(\alpha-\beta)$

$\therefore \cos^2\alpha+\cos^2\beta-2\cos\alpha\cos\beta+\sin^2\alpha+\sin^2\beta-2\sin\alpha\sin\beta$

$=\cos^2(\alpha-\beta)-2\cos(\alpha-\beta)+1+\sin^2(\alpha-\beta)$

$\therefore 2-2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)=2-2\cos(\alpha-\beta)$

$\therefore \cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

除以 $\cos\alpha\cos\beta$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

【例 1】已知$\cos\alpha=-\frac{3}{5}$，且$\alpha\in(\frac{\pi}{2},\pi)$，则$\cos(\frac{\pi}{4}-\alpha)=$______.

$$\cos(\frac{\pi}{4}-a)=\cos\frac{\pi}{4}\cos a+\sin a\sin\frac{\pi}{4}$$

$$=-\frac{3}{5}\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\sin a$$

$$=-\frac{3}{5}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\times\sqrt{1-\cos^2 a}$$

$$=\frac{\sqrt{2}}{2}\times(-\frac{3}{5}+\frac{4}{5})$$

$$=\frac{\sqrt{2}}{10}$$

$180 \quad 270$

【例 2】已知 $\sin\alpha=-\frac{2}{3}$， $\cos\beta=\frac{3}{4}$， 且 $\alpha\in(\pi,\frac{3\pi}{2})$， $\beta\in(\frac{3\pi}{2},2\pi)$， 则 $\cos(\beta-\alpha)=$______.

$$\cos(\beta-\alpha)=\cos\alpha\cos\beta+\sin\beta\sin\alpha$$

$$=-\sqrt{1-(-\tfrac{2}{3})^2}\times\frac{3}{4}+-\sqrt{1-(\tfrac{3}{4})^2}\times(-\tfrac{2}{3})$$

$$=-\sqrt{\frac{5}{9}}\times\frac{3}{4}+\sqrt{\frac{7}{16}}\times\frac{2}{3}$$

$$=-\frac{\sqrt{5}}{3}\times\frac{3}{4}+\frac{\sqrt{7}}{4}\times\frac{2}{3}$$

$$=-\frac{\sqrt{5}}{4}+\frac{\sqrt{7}}{6}$$

$$=\frac{2\sqrt{7}-3\sqrt{5}}{12}$$

【例 3】已知 $\cos\theta=-\frac{3}{5}$， 且 θ 为钝角， 则 $\sin(\theta+\frac{\pi}{3})=\underline{\frac{4-3\sqrt{3}}{10}}$.

$\frac{1}{2} \quad \frac{\sqrt{3}}{2}$

$$\sin(\theta+\frac{\pi}{3})=\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}$$

$$=\frac{4}{5}\times\frac{1}{2}+(-\tfrac{3}{5})\times\frac{\sqrt{3}}{2}$$

$$=\frac{2}{5}-\frac{3\sqrt{3}}{10}=\frac{4-3\sqrt{3}}{10}$$

$$\sin\theta=\sqrt{1-(-\tfrac{3}{5})^2}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

（1） $\sin 70^\circ \sin 50^\circ - \cos 70^\circ \sin 40^\circ =$______；

（2） $\sin(54^\circ - x)\cos(36^\circ + x) + \cos(54^\circ - x)\sin(36^\circ + x) =$______；

(2) 原式 $= \sin(54^\circ - x + 36^\circ + x) = \sin 90^\circ = 1$

（3） $\cos\frac{\pi}{12} + \sqrt{3}\sin\frac{\pi}{12} =$______；

（4） $\tan 15^\circ + \tan 30^\circ + \tan 15^\circ \tan 30^\circ =$______.

(1) 原式 $= \sin 70^\circ \sin 50^\circ - \cos 70^\circ \cos 50^\circ = -(\cos 70^\circ \cos 50^\circ - \sin 70^\circ \sin 50^\circ) = -\cos(70^\circ + 50^\circ) = \frac{1}{2}$

(2) 原式 $= \sin(54^\circ - x)^2 + \cos(54^\circ - x)^2$

$= (\sin 54 \cos x - \cos 54^\circ \sin x)^2 + (\cos 54^\circ \cdot \cos x + \sin 54^\circ \sin x)^2$

$= \sin^2 54^\circ \cos^2 x + \cos^2 54^\circ \sin^2 x + \cos^2 54^\circ \cos^2 x + \sin^2 54^\circ \sin^2 x$

$= (\sin^2 54^\circ + \cos^2 54^\circ)\cos^2 x + (\cos^2 54^\circ + \sin^2 54^\circ)\sin^2 x$

$= 1$