1. $\sin 2\alpha = \sin(\alpha+\alpha) = 2\sin\alpha\cos\alpha$

2. $\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1-2\sin^2\alpha$

$\cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$ $\quad \sin^2\alpha = \dfrac{1-\cos 2\alpha}{2}$ 降次

3. $\tan 2\alpha = \tan(\alpha+\alpha) = \dfrac{\tan\alpha + \tan\alpha}{1-\tan\alpha\cdot\tan\alpha} = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

【例 1】已知 $\sin(\alpha-\pi)=\dfrac{3}{5}$，则 $\cos 2\alpha =$______.

$\sin(\alpha-\pi) = \sin(\pi+\alpha) = -\sin\alpha = \frac{3}{5}$

$\sin\alpha = -\frac{3}{5}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\sqrt{1-\sin^2\alpha}\right)^2 - \sin^2\alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

【例 2】已知$\sin\alpha=-\frac{3\sqrt{10}}{10}$，且$\alpha$是第四象限的角，则$\tan 2\alpha=$______.

$\cos\alpha=\sqrt{1-\sin^2\alpha}=\frac{\sqrt{10}}{10}$

$\tan\alpha=-3$

$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{-6}{-8}=\frac{3}{4}$

【例 3】已知$\sqrt{2}\sin 2\alpha=-\sin\alpha$，且$\frac{\pi}{2}<\alpha<\pi$，则$\tan\alpha=$ $-\sqrt{7}$.

$2\sqrt{2}\sin\alpha\cos\alpha=-\sin\alpha$

$2\sqrt{2}\cos\alpha=-1$

$\cos\alpha=-\frac{1}{2\sqrt{2}}=-\frac{\sqrt{2}}{4}$

$\sin\alpha=\sqrt{1-(\frac{\sqrt{2}}{4})^2}=\sqrt{\frac{7}{8}}=\frac{\sqrt{7}}{2\sqrt{2}}=\frac{\sqrt{14}}{4}$

$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\sqrt{14}}{4}\times(-\frac{4}{\sqrt{2}})=-\sqrt{7}$

【例 4】已知 $\tan\alpha=\frac{1}{3}$， $\tan\beta=-\frac{1}{7}$， 则 $\tan(2\alpha-\beta)=$ __1__.

$$\tan(2\alpha-\beta)=\frac{\tan 2\alpha-\tan\beta}{1+\tan 2\alpha\tan\beta}=\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}=\frac{21+4}{28-3}=1$$

$$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{\frac{2}{3}}{\frac{8}{9}}=\frac{2}{3}\times\frac{9}{8}=\frac{3}{4}$$

（1） $\dfrac{\tan 15^\circ}{1-\tan^2 15^\circ}$；

（2） $\dfrac{\sqrt{1+\sin 20^\circ}+\sqrt{1-\sin 20^\circ}}{\cos 10^\circ}$；

（3） $\dfrac{\sqrt{1-2\sin 20^\circ\cos 20^\circ}}{2\cos^2 10^\circ-\sqrt{1-\cos^2 160^\circ}-1}$；

（4） $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$.

(1) $\frac{\tan 15^\circ}{1-\tan^2 15^\circ}$;

(2) $\frac{\sqrt{1+\sin 20^\circ}+\sqrt{1-\sin 20^\circ}}{\cos 10^\circ}$;

(3) $\frac{\sqrt{1-2\sin 20^\circ\cos 20^\circ}}{2\cos^2 10^\circ-\sqrt{1-\cos^2 160^\circ}-1}$;

(4) $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$.

$\frac{2\tan 15^\circ}{1-\tan^2 15^\circ}=\frac{1}{2}\times\tan 30^\circ=\frac{1}{2}\times\frac{\sqrt{3}}{3}=\frac{\sqrt{3}}{6}$

$\frac{2\cos 10^\circ}{\cos 10^\circ}=2$

$\sin^2 10^\circ+\sin(10^\circ+10^\circ)=\cos^2 10^\circ+\sin^2 10^\circ+2\cos 10^\circ\sin 10^\circ$

$=(\cos 10^\circ+\sin 10^\circ)^2$

(3) $\frac{\sqrt{1-2\sin 20^\circ\cos 20^\circ}}{2\cos^2 10^\circ-\sqrt{1-\cos^2 160^\circ}-1}=\frac{\cos 20^\circ-\sin 20^\circ}{\cos 20^\circ-\sin 20^\circ}=1$

$\sqrt{1-2\sin 20^\circ\cos 20^\circ}=\sqrt{(\sin 20^\circ-\cos 20^\circ)^2}=|\sin 20^\circ-\cos 20^\circ|=\cos 20^\circ-\sin 20^\circ$

$\sqrt{1-\cos^2 160^\circ}=\sqrt{\sin^2 20^\circ+\cos^2 20^\circ-\cos^2 20^\circ}=\sqrt{\sin^2 20^\circ}$

$2\cos^2 10^\circ=\frac{1+\cos 20^\circ}{2}\times 2=1+\cos 20^\circ$

(4) 原式 $= \dfrac{\sin\frac{\pi}{11}\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cdots\cos\frac{5\pi}{11}}{\sin\frac{\pi}{11}} = \dfrac{\sin\frac{2\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}}{2\sin\frac{\pi}{11}} = \dfrac{\sin\frac{4\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}}{4\sin\frac{\pi}{11}}$

$= \dfrac{\sin\frac{8\pi}{11}\cos\frac{3\pi}{11}\cos\frac{5\pi}{11}}{8\sin\frac{\pi}{11}} = \dfrac{\sin\frac{3\pi}{11}\cos\frac{3\pi}{11}\cos\frac{5\pi}{11}}{8\sin\frac{\pi}{11}} = \dfrac{\sin\frac{6\pi}{11}\cos\frac{5\pi}{11}}{16\sin\frac{\pi}{11}} = \dfrac{\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{16\sin\frac{\pi}{11}} = \dfrac{\sin\frac{10\pi}{11}}{32\sin\frac{\pi}{11}}$