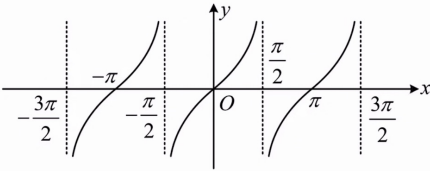




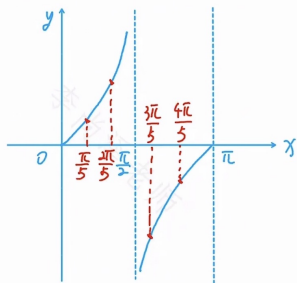
函数	$y = \tan x$	$y = A \tan(\omega x + \varphi) (A > 0, \omega > 0)$
定义域	$\{x \mid x \neq k\pi + \frac{\pi}{2}, k \in \mathbf{Z}\}$	$\{x \mid \omega x + \varphi \neq k\pi + \frac{\pi}{2}, k \in \mathbf{Z}\}$
值域	$\mathbf{R}$	$\mathbf{R}$
最小正周期	π	$\frac{\pi}{\omega}$
奇偶性	奇函数	当 $\varphi = \frac{k\pi}{2} (k \in \mathbf{Z})$ 时为奇函数，否则为非奇非偶函数
增区间	$(k\pi - \frac{\pi}{2}, k\pi + \frac{\pi}{2}) (k \in \mathbf{Z})$	$(\frac{1}{\omega}(k\pi - \frac{\pi}{2} - \varphi), \frac{1}{\omega}(k\pi + \frac{\pi}{2} - \varphi)) (k \in \mathbf{Z})$
对称中心	$(\frac{k\pi}{2}, 0) (k \in \mathbf{Z})$	$(\frac{1}{\omega}(\frac{k\pi}{2} - \varphi), 0) (k \in \mathbf{Z})$



【例 1】函数 $y = \tan(2x - \frac{\pi}{6})$ 的定义域为 $\{x | x \neq \frac{k\pi}{2} + \frac{\pi}{3}, k \in \mathbb{Z}\}$.

$$2x - \frac{\pi}{6} \neq k\pi + \frac{\pi}{2} \quad \therefore x \neq \frac{k\pi}{2} + \frac{\pi}{3}$$

【例 2】若 $a = \tan \frac{\pi}{5}$, $b = \tan \frac{2\pi}{5}$, $c = \tan \frac{3\pi}{5}$, $d = \tan \frac{4\pi}{5}$, 则 a 、 b 、 c 、 d 的大小关系为_____.



$$b > a > d > c$$

【例 3】函数 $y = \tan(2x - \frac{\pi}{6})$ 的单调递增区间是 $[\frac{k\pi}{2} - \frac{\pi}{6}, \frac{k\pi}{2} + \frac{\pi}{3}]$

$$k\pi - \frac{\pi}{2} < 2x - \frac{\pi}{6} < k\pi + \frac{\pi}{2}$$

$$k\pi - \frac{\pi}{3} < 2x < k\pi + \frac{2}{3}\pi$$

$$\frac{k\pi}{2} - \frac{\pi}{6} < x < \frac{k\pi}{2} + \frac{\pi}{3}$$

【例 4】函数 $y = \tan(\frac{\pi}{4} - x)$ 的单调递减区间是_____.

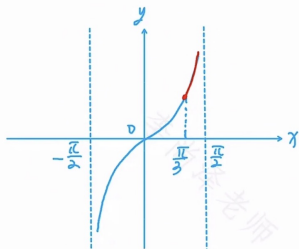
$$y = \tan(\frac{\pi}{4} - x) = -\tan(x - \frac{\pi}{4})$$

$$k\pi - \frac{\pi}{2} < x - \frac{\pi}{4} < k\pi + \frac{\pi}{2}$$

$$k\pi - \frac{\pi}{4} < x < k\pi + \frac{3}{4}\pi$$

题组三：解正切不等式

【例 5】不等式 $\tan x \geq \sqrt{3}$ 的解集是 $[k\pi + \frac{\pi}{3}, k\pi + \frac{\pi}{2})$, $k \in \mathbb{Z}$.



【例 6】函数 $y = \sqrt{1 - \tan(2x + \frac{\pi}{4})}$ 的定义域是_____.

$$1 - \tan(2x + \frac{\pi}{4}) \geq 0$$

$$\tan(2x + \frac{\pi}{4}) \leq 1$$

$$\text{当 } 2x + \frac{\pi}{4} = \frac{\pi}{4} \text{ 时, } \tan(2x + \frac{\pi}{4}) = 1$$

$$k\pi - \frac{\pi}{2} < 2x + \frac{\pi}{4} \leq \frac{\pi}{4} + k\pi$$

$$k\pi - \frac{3}{4}\pi < 2x \leq k\pi$$

$$\frac{k\pi}{2} - \frac{3}{8}\pi < x \leq \frac{k\pi}{2}$$

题组四：周期与对称性

【例 7】函数 $f(x) = \tan(\frac{\pi}{3} - 3x)$ 的最小正周期是_____.

$$y = A \tan(\omega x + \varphi) \quad T = \frac{\pi}{|\omega|}$$

$$T = \frac{\pi}{|-3|}$$

【例 8】（多选）函数 $f(x) = \tan(3x + \frac{\pi}{4})$ 的对称中心有（ ）

- (A) $(\frac{\pi}{12}, 0)$ (B) $(\frac{\pi}{6}, 0)$ (C) $(-\frac{\pi}{4}, 0)$ (D) $(\frac{\pi}{3}, 0)$

【例 9】已知函数 $f(x) = A \tan(\omega x + \varphi)$ ($\omega > 0, |\varphi| < \frac{\pi}{2}$) 的部分图象如图所示，则 $f(\frac{\pi}{24}) =$ _____.

$$\frac{T}{2} = \frac{3\pi}{8} - \frac{\pi}{8} = \frac{\pi}{4} \quad \therefore T = \frac{\pi}{2} \quad \therefore \omega = 2$$

$$f(x) = A \tan(2x + \varphi) \quad f(\frac{3\pi}{8}) = A \tan(\frac{3\pi}{4} + \varphi) = 0$$

$$\frac{3\pi}{4} + \varphi = k\pi \quad \varphi = k\pi - \frac{3\pi}{4} \quad (k \in \mathbb{Z}) \quad \text{又 } |\varphi| < \frac{\pi}{2} \quad \therefore \varphi = \frac{\pi}{4}$$

$$f(0) = A \tan \varphi = A = 1 \quad \therefore f(x) = \tan(2x + \frac{\pi}{4})$$

$$f(\frac{\pi}{24}) = \tan(\frac{\pi}{12} + \frac{\pi}{4}) = \tan \frac{\pi}{3} = \sqrt{3}.$$

