$\sin x+\cos x$. $\sin x\cos x$. $\sin x-\cos x$

1. $(\sin x+\cos x)^2=\sin^2 x+\cos^2 x+2\sin x\cos x=1+2\sin x\cos x$

2. $(\sin x-\cos x)^2=1-2\sin x\cos x$

3. $(\sin x+\cos x)^2+(\sin x-\cos x)^2=2$

【例 1】已知$\cos\theta+\sin\theta=-\frac{1}{2}$，则$\sin\theta\cos\theta=$ $\underline{-\frac{3}{8}}$.

$(\cos\theta+\sin\theta)^2=\frac{1}{4}=1+2\sin\theta\cos\theta$

$2\sin\cos\theta=-\frac{3}{4}$

【例 2】已知α为第四象限的角，且$\sin\alpha+\cos\alpha=\frac{1}{5}$，则$\frac{1}{\cos^2\alpha-\sin^2\alpha}=$ $\underline{\frac{25}{7}}$.

$0<a<-90^\circ$ $(\sin a+\cos a)^2+(\sin a-\cos a)^2=2$

$\frac{1}{25}+(\sin a-\cos a)^2=2$

$(\sin a-\cos a)^2=\frac{49}{25}$

$\sin a-\cos a=\frac{7}{5}$

【例 3】已知$\alpha\in(0,\pi)$，$\sin\alpha-\cos\alpha=\frac{\sqrt{3}+1}{2}$，则$\tan\alpha=$ $\underline{-\frac{\sqrt{3}}{3}}$ 或 $-\sqrt{3}$

$\because 0<a<180^\circ$, $\sin a>0$, $(\sin a-\cos a)^2=1-2\sin a\cos a=1+\frac{\sqrt{3}}{2}$

$\therefore \sin a\cos a=\frac{\sqrt{3}}{4}$

$\sin a\cos a=\frac{\sin a\cos a}{\sin^2 a+\cos^2 a}=\frac{\tan a}{1+\tan^2 a}=-\frac{\sqrt{3}}{4}\implies$ ① $\cos a>0$, $\sin a>0$ ×

（第一象限，$\frac{\pi}{2}>a>0$）

② $\cos a<0$, $\sin a>0$ ✓ $\implies \frac{\pi}{2}<a<\pi$)

$\sqrt{3}+\sqrt{3}\tan^2 a=4\tan a$

$3+3\tan^2 a=-4\sqrt{3}\tan a$

$3\tan^2 a+4\tan a+3=0$

$(3\tan a+\sqrt{3})(\tan a+\sqrt{3})=0$

$\tan a=-\sqrt{3}$ 或 $3\tan a=-\sqrt{3}$