



$$(1) \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$(2) \sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$(3) \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad (1)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad (2)$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad (3)$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \quad (4)$$

$$(1) \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$(2) \sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$(3) \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$(4) \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

【例 2】化简：(1) $\sin(\frac{\pi}{4}-x)\sin(\frac{\pi}{4}+x)$; (2) $\sin(x+\frac{\pi}{3})\cos(\frac{\pi}{6}+x)$.

$$\begin{aligned}(1) \text{ 原式} &= \frac{1}{2} \{ \cos[(\frac{\pi}{4}-x)-(\frac{\pi}{4}+x)] - \cos[(\frac{\pi}{4}+x)+(\frac{\pi}{4}-x)] \} \\ &= \frac{1}{2} \{ \cos \pi - \cos \frac{\pi}{2} \} \\ &= \frac{\cos \pi}{2}\end{aligned}$$

$$\begin{aligned}(2) \text{ 原式} &= \frac{1}{2} \{ \sin[(x+\frac{\pi}{3})+(\frac{\pi}{6}+x)] + \sin[(x+\frac{\pi}{3})-(\frac{\pi}{6}+x)] \} \\ &= \frac{1}{2} \{ \sin(\frac{\pi}{2}+2x) + \sin \frac{\pi}{6} \} \\ &= \frac{1}{2} (\cos 2x + \frac{1}{2})\end{aligned}$$