



(1) 公式二: $\sin(\pi + \alpha) = \underline{-\sin \alpha}$, $\cos(\pi + \alpha) = \underline{-\cos \alpha}$, $\tan(\pi + \alpha) = \underline{\tan \alpha}$.

(2) 公式三: $\sin(-\alpha) = \underline{-\sin \alpha}$, $\cos(-\alpha) = \underline{\cos \alpha}$, $\tan(-\alpha) = \underline{-\tan \alpha}$.

(3) 公式四: $\sin(\pi - \alpha) = \underline{\sin \alpha}$, $\cos(\pi - \alpha) = \underline{-\cos \alpha}$, $\tan(\pi - \alpha) = \underline{-\tan \alpha}$.

(4) 公式五: $\sin(\frac{\pi}{2} - \alpha) = \underline{\cos \alpha}$, $\cos(\frac{\pi}{2} - \alpha) = \underline{\sin \alpha}$.

(5) 公式六: $\sin(\frac{\pi}{2} + \alpha) = \underline{\cos \alpha}$, $\cos(\frac{\pi}{2} + \alpha) = \underline{-\sin \alpha}$. 假设

2. 诱导公式统一口诀: 奇变偶不变, 符号看象限 (α 为锐角).

$\frac{1}{2}\pi$ 的几倍

$\downarrow$
 $\sin \leftrightarrow \cos$
 $\cos \leftrightarrow \sin$

看括号内的角度

(1) $\sin 960^\circ$; (2) $\cos \frac{23\pi}{6}$; (3) $\sin(-\frac{7\pi}{3})$.

(1) $\sin 960^\circ = \sin(960^\circ - 720^\circ) = \sin 240^\circ = \sin(\pi + 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

(2) $\cos \frac{23\pi}{6} = \cos(\frac{23}{6}\pi - 4\pi) = \cos(-\frac{1}{6}\pi) = \cos \frac{1}{6}\pi = \cos 30^\circ = \frac{\sqrt{3}}{2}$

(3) $\sin(-\frac{7}{3}\pi) = \sin(-\frac{7}{3}\pi + 2\pi) = \sin(-\frac{1}{3}\pi) = -\sin \frac{1}{3}\pi = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

$$(1) \frac{\sqrt{1 + \cos 100^\circ \sin 170^\circ}}{\cos 370^\circ + \sqrt{1 - \sin^2 170^\circ}};$$

$$(2) \cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{5\pi}{7} + \cos \frac{6\pi}{7} = 0$$

$$(3) \cos^2 1^\circ + \cos^2 3^\circ + \cos^2 5^\circ + \dots + \cos^2 89^\circ.$$

$$(1) \frac{\sqrt{1 + \cos 100^\circ \sin 170^\circ}}{\cos 370^\circ + \sqrt{1 - \sin^2 170^\circ}} = \frac{\cos 10^\circ}{\cos 10^\circ + \cos 10^\circ} = \frac{1}{2}$$

$$\begin{aligned} \sqrt{1 + \cos 100^\circ \sin 170^\circ} &= \sqrt{1 + \cos(\frac{10}{2} + 10^\circ) \sin(180^\circ - 10^\circ)} = \sqrt{1 + \sin 10^\circ \cdot (-\sin 10^\circ)} = \sqrt{1 - \sin^2 10^\circ} = \sqrt{\cos^2 10^\circ} \quad (\sin^2 10^\circ + \cos^2 10^\circ = 1) = \cos 10^\circ \\ \sqrt{1 - \sin^2 170^\circ} &= \sqrt{\cos^2 170^\circ} = \cos 170^\circ = \cos(180^\circ - 10^\circ) = -\cos 10^\circ \\ \cos 370^\circ &= \cos(360^\circ + 10^\circ) = \cos 10^\circ \end{aligned}$$

$$(2) \cos \frac{6}{7}\pi = \cos(\pi - \frac{1}{7}\pi) = -\cos \frac{1}{7}\pi$$

$$\cos \frac{5}{7}\pi = -\cos \frac{2}{7}\pi$$

$$\cos \frac{4}{7}\pi = -\cos \frac{3}{7}\pi$$

$$(3) \cos^2 89^\circ = \cos^2(\frac{\pi}{2} - 1^\circ) = \sin^2 1^\circ + \cos^2 1^\circ = 1$$

$$\therefore \cos^2 1^\circ + \cos^2 3^\circ + \dots + \cos^2 89^\circ = 22 + \cos^2 45^\circ = 22 + (\frac{\sqrt{2}}{2})^2 = 22\frac{1}{2}$$

【例 4】已知 $\sin(\frac{\pi}{3}-\alpha)=\frac{1}{3}$, 则 $\cos(\frac{\pi}{6}+\alpha)=$ _____.

配凑: $\cos(\frac{\pi}{2}-\beta)=\sin\beta$

$$(-\frac{\pi}{2}-\alpha)+(\frac{\pi}{6}+\alpha)=\frac{\pi}{2}$$

$$\cos(\frac{\pi}{6}+\alpha)=\cos[\frac{\pi}{2}-(\frac{\pi}{3}-\alpha)]=\sin(\frac{\pi}{3}-\alpha)=\frac{1}{3}$$

【例 5】已知 $\tan(\frac{\pi}{6}+\alpha)=2$, 则 $\tan(\frac{5\pi}{6}-\alpha)=$ _____.

$\tan(\pi-\alpha)=-\tan\alpha$

$$\frac{\pi}{6}+\alpha+\frac{5}{6}\pi-\alpha=\pi$$

$$\tan(\frac{5}{6}\pi-\alpha)=\tan[\pi-(\frac{1}{6}\pi+\alpha)]=-\tan(\frac{1}{6}\pi+\alpha)=-2$$

【例 6】已知 $\cos(\frac{a}{75^\circ}+\alpha)=\frac{1}{3}$, 且 α 为第三象限的角, 则 $\sin(\alpha-105^\circ)=$ _____.

锐角

$$\frac{a}{75^\circ}+\alpha-(\alpha-105^\circ)=180^\circ=\pi$$

$$\sin(\alpha-105^\circ)=\sin[(75^\circ+\alpha)-180^\circ]=-\sin(75^\circ+\alpha)$$

$$\because \cos(75^\circ+\alpha)=\frac{1}{3} \quad \alpha \text{ 为第三象限角} \quad \alpha+75^\circ \text{ 为第三或第四象限角}$$

$$\therefore \sin(75^\circ+\alpha)=-\sqrt{1-\cos^2(75^\circ+\alpha)}=-\frac{2\sqrt{2}}{3}$$

$$\sin(\alpha-105^\circ)=\frac{2}{3}\sqrt{2}$$