$\cos\alpha = \cdots\cdots \qquad \cos(\alpha+\beta) = \cdots\cdots \qquad \sin\beta = ?$

$\sin\beta = \sin[(\alpha+\beta)-\alpha] = \sin(\alpha+\beta)\cos\alpha - \cos(\alpha+\beta)\sin\alpha$

【例 1】已知α为锐角，且$\cos(\alpha+\frac{\pi}{3})=-\frac{3}{5}$，则$\sin\alpha=$______.

$\sin\alpha = \sin[(\alpha+\frac{\pi}{3})-\frac{\pi}{3}] = \sin(\alpha+\frac{\pi}{3})\cos\frac{\pi}{3} - \cos(\alpha+\frac{\pi}{3})\sin\frac{\pi}{3}$

$\alpha\in(0,\frac{\pi}{2}) \qquad \alpha+\frac{\pi}{3}\in(\frac{\pi}{3},\frac{5\pi}{6})$

$\sin(\alpha+\frac{\pi}{3}) = \sqrt{1-\cos^2(\alpha+\frac{\pi}{3})} = \frac{4}{5}$

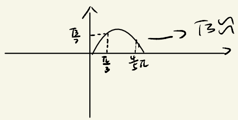
$\therefore \sin\alpha = \frac{4}{5}\times\frac{1}{2}+\frac{3}{5}\times\frac{\sqrt{3}}{2} = \frac{4+3\sqrt{3}}{10}$

【例 2】已知α、β均为锐角，$\cos(\alpha+\beta)=-\dfrac{5}{13}$，$\sin(\beta+\dfrac{\pi}{3})=\dfrac{4}{5}$，则$\cos(\alpha-\dfrac{\pi}{3})=$______.

$$\cos(a-\tfrac{\pi}{3})=\cos[(a+\beta)-(\beta+\tfrac{\pi}{3})]=\cos(a+\beta)\cos(\beta+\tfrac{\pi}{3})+\sin(a+\beta)\sin(\beta+\tfrac{\pi}{3})$$

$$\sin(a+\beta)=\sqrt{1-\cos^2(a+\beta)}=\sqrt{1-(-\tfrac{5}{13})^2}=\sqrt{1-\tfrac{25}{169}}=\sqrt{\tfrac{144}{169}}=\tfrac{12}{13}$$

$\beta+\frac{\pi}{3}\in(\frac{\pi}{3},\frac{5}{6}\pi)$，且符合$\sin(\beta+\frac{\pi}{3})=\frac{4}{5}$



$\longrightarrow$ $\sqrt{3}$约1.732，$\frac{\sqrt{3}}{2}$约$0.866>\frac{4}{5}$

$\therefore \beta+\frac{\pi}{3}$在第二象限

$$\cos(\beta+\tfrac{\pi}{3})=-\sqrt{1-\sin^2(\beta+\tfrac{\pi}{3})}=-\sqrt{1-\tfrac{16}{25}}=-\tfrac{3}{5}$$

$$\cos(a-\tfrac{\pi}{3})=-\tfrac{5}{13}\times(-\tfrac{3}{5})+\tfrac{12}{13}\times\tfrac{4}{5}=\tfrac{3}{13}+\tfrac{48}{65}=\tfrac{63}{65}$$

【例 3】已知$\tan(\alpha+\beta)=\dfrac{3}{5}$，$\tan(\beta-\dfrac{\pi}{3})=\dfrac{1}{3}$，则$\tan(\alpha+\dfrac{\pi}{3})=$______.

$$\tan(a+\tfrac{\pi}{3})=\tan[(a+\beta)-(\beta-\tfrac{\pi}{3})]=\frac{\tan(a+\beta)-\tan(\beta-\frac{1}{3})}{1+\tan(a+\beta)\cdot\tan(\beta-\frac{\pi}{3})}$$

$$=\frac{\frac{3}{5}-\frac{1}{3}}{1+\frac{3}{5}\times\frac{1}{3}}$$

$$=\frac{\frac{4}{15}}{\frac{6}{5}}=\frac{2}{9}$$

【例 4】已知$\frac{\pi}{2}<\beta<\alpha<\frac{3\pi}{4}$，$\cos(\alpha-\beta)=\frac{12}{13}$，$\sin(\alpha+\beta)=-\frac{3}{5}$，则$\sin 2\alpha=$______.

$\because \frac{\pi}{2}<\beta<\alpha<\frac{3}{4}\pi$

$\therefore \alpha-\beta\in(0,\frac{3}{4}\pi)$

$\alpha+\beta\in(\pi,\frac{5}{4}\pi)$

$\sin 2\alpha=\sin[(\alpha-\beta)+(\alpha+\beta)]=\sin(\alpha-\beta)\cos(\alpha+\beta)+\cos(\alpha-\beta)\sin(\alpha+\beta)$

$\sin(\alpha-\beta)=\sqrt{1-\cos^2(\alpha-\beta)}=\sqrt{1-\frac{144}{169}}=\frac{5}{13}$

$\cos(\alpha+\beta)=-\sqrt{1-(-\frac{3}{5})^2}=-\frac{4}{5}$

$\sin 2\alpha=\frac{5}{13}\times(-\frac{4}{5})+\frac{12}{13}\times(-\frac{3}{5})$

$=-\frac{4}{13}-\frac{36}{65}$

$=-\frac{56}{65}$

【例 5】已知α、β均为钝角，且$\sin\alpha=\frac{\sqrt{5}}{5}$，$\cos\beta=-\frac{3\sqrt{10}}{10}$，则$\alpha+\beta=$（ D ）

（A）$\frac{3\pi}{2}$　（B）$\frac{5\pi}{4}$　（C）$\frac{5\pi}{4}$或$\frac{7\pi}{4}$　（D）$\frac{7\pi}{4}$

$\alpha,\beta\in(\frac{\pi}{2},\pi)$ $\alpha+\beta\in(\pi,2\pi)$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cdot\cos\alpha$

$\cos\alpha=-\sqrt{1-(\frac{\sqrt{5}}{5})^2}=-\frac{\sqrt{20}}{5}=-\frac{2}{5}\sqrt{5}$

$\sin\beta=\sqrt{1-(-\frac{3}{10}\sqrt{10})^2}=\frac{\sqrt{10}}{10}$

$\sin(\alpha+\beta)=\frac{\sqrt{5}}{5}\times(-\frac{3}{10}\sqrt{10})+-\frac{2}{5}\sqrt{5}\times\frac{\sqrt{10}}{10}$

$=-\frac{3}{50}\sqrt{50}-\frac{2}{50}\sqrt{50}$

$=-\frac{1}{10}\sqrt{50}$

$=-\frac{\sqrt{2}}{2}$

$\sin\frac{7}{4}\pi=\sin(2\pi-\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

【例 6】已知α、β均为锐角，且$\cos(\alpha+\beta)=\dfrac{\sqrt{5}}{5}$，$\sin(\alpha-\beta)=\dfrac{\sqrt{10}}{10}$，则$\beta=$ $\underline{\dfrac{\pi}{8}\ (22.5^\circ)}$.

$\alpha, \beta \in (0, \frac{\pi}{2})$ $\quad \alpha+\beta \in (0, \frac{\pi}{2})$ $\quad \alpha-\beta \in (0, \frac{\pi}{2})$ $\quad 2\beta \in (0, \frac{\pi}{2})$

$\sin 2\beta = \sin[(\alpha+\beta)-(\alpha-\beta)] = \sin(\alpha+\beta)\cos(\alpha-\beta) - \sin(\alpha-\beta)\cos(\alpha+\beta)$

$\sin(\alpha+\beta) = \sqrt{1-(\frac{\sqrt{5}}{5})^2} = \sqrt{\frac{20}{25}} = \frac{2}{5}\sqrt{5}$

$\cos(\alpha-\beta) = \sqrt{1-(\frac{\sqrt{10}}{10})^2} = \frac{3}{10}\sqrt{10}$

$\sin 2\beta = \frac{2}{5}\sqrt{5} \times \frac{3}{10}\sqrt{10} - \frac{\sqrt{5}}{5} \times \frac{\sqrt{10}}{10}$

$= \frac{3}{25}\sqrt{50} - \frac{1}{50}\sqrt{50}$

$= \frac{3}{5}\sqrt{2} - \frac{1}{10}\sqrt{2}$

$= \frac{\sqrt{2}}{2}$

$2\beta = \frac{\pi}{4}$

$\beta = \frac{\pi}{8}$