



$$(1) \frac{\sin 47^\circ - \sin 17^\circ \cos 30^\circ}{\cos 17^\circ};$$

$$(2) \tan 12^\circ + \tan 33^\circ + \tan 12^\circ \tan 33^\circ;$$

$$(3) (\tan 10^\circ - \sqrt{3}) \cdot \frac{\cos 10^\circ}{\sin 50^\circ};$$

$$(4) \frac{2 \cos 10^\circ - \sin 20^\circ}{\cos 20^\circ}.$$

$$(1) \tan 45^\circ = \tan(12^\circ + 33^\circ) = \frac{\tan 12^\circ + \tan 33^\circ}{1 - \tan 12^\circ \tan 33^\circ} = 1$$

$$\therefore \tan 12^\circ + \tan 33^\circ = 1 - \tan 12^\circ \tan 33^\circ$$

$$\therefore \tan 12^\circ + \tan 33^\circ + \tan 12^\circ \tan 33^\circ = 1$$

$$(1) \frac{\sin 47^\circ - \sin 17^\circ \cos 30^\circ}{\cos 17^\circ} = \frac{\sin 30^\circ \cos 17^\circ + \cos 30^\circ \sin 17^\circ - \sin 17^\circ \cos 30^\circ}{\cos 17^\circ} = \sin 30^\circ = \frac{1}{2}$$

$$\begin{aligned} (3) (\tan 10^\circ - \sqrt{3}) \cdot \frac{\cos 10^\circ}{\sin 50^\circ} &= \frac{\tan 10^\circ \cdot \cos 10^\circ - \sqrt{3} \cos 10^\circ}{\sin 50^\circ} = \frac{\sin 10^\circ - \sqrt{3} \cos 10^\circ}{\sin 50^\circ} \\ &= \frac{2(\frac{1}{2} \sin 10^\circ - \frac{\sqrt{3}}{2} \cos 10^\circ)}{\sin 50^\circ} = \frac{2(\cos 60^\circ \sin 10^\circ - \cos 10^\circ \sin 60^\circ)}{\sin 50^\circ} \\ &= \frac{-2 \sin 50^\circ}{\sin 50^\circ} = -2 \end{aligned}$$

$$\begin{aligned}
 (4) \quad \frac{2\cos 10^\circ - \sin 20^\circ}{\cos 20^\circ} &= \frac{2(\cos(30^\circ - 20^\circ) - \sin 20^\circ)}{\cos 20^\circ} \\
 &= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ + \frac{1}{2}\sin 20^\circ\right) - \sin 20^\circ}{\cos 20^\circ} \\
 &= \frac{\sqrt{3}\cos 20^\circ + \sin 20^\circ - \sin 20^\circ}{\cos 20^\circ} \\
 &= \sqrt{3}
 \end{aligned}$$

【例 2】计算：(1) $[2\sin 50^\circ + \sin 10^\circ(1 + \sqrt{3}\tan 10^\circ)] \times \sqrt{2\sin^2 80^\circ}$ ；(2) $\frac{\sin 7^\circ + \cos 15^\circ \sin 8^\circ}{\cos 7^\circ - \sin 15^\circ \sin 8^\circ}$ 。

$$\begin{aligned}
 (1) &= [2\sin(60^\circ - 10^\circ) + \sin 10^\circ + \sqrt{3} \frac{\sin^2 10^\circ}{\cos 10^\circ}] \times \sqrt{2} \cos 10^\circ \\
 &= [2 \times (\frac{\sqrt{3}}{2}\cos 10^\circ - \frac{1}{2}\sin 10^\circ) + \sin 10^\circ + \sqrt{3} \frac{\sin^2 10^\circ}{\cos 10^\circ}] \times \sqrt{2} \cos 10^\circ \\
 &= (\sqrt{3}\cos 10^\circ - \sin 10^\circ + \sqrt{3} \frac{\sin^2 10^\circ}{\cos 10^\circ}) \times \sqrt{2} \cos 10^\circ \\
 &= \sqrt{6} \cos^2 10^\circ + \sqrt{6} \sin^2 10^\circ \\
 &= \sqrt{6}
 \end{aligned}$$

$$12) = \frac{\sin(15-8)^\circ + \cos 15^\circ \sin 8^\circ}{\cos(15-7)^\circ - \sin 15^\circ \sin 8^\circ}$$

$$= \frac{\sin 15^\circ \cos 8^\circ - \cos 15^\circ \sin 8^\circ + \cos 15^\circ \sin 8^\circ}{\cos 15^\circ \cos 8^\circ + \sin 15^\circ \sin 8^\circ - \sin 15^\circ \sin 8^\circ}$$

$$= \frac{\sin 15^\circ \cos 8^\circ}{\cos 15^\circ \cos 8^\circ}$$

$$= \tan 15^\circ$$

$$= \tan (60^\circ - 45^\circ)$$

$$= \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} = \frac{(\sqrt{3}-1)^2}{2} = 2 - \sqrt{3}$$

【例 3】 $(1 + \tan 1^\circ)(1 + \tan 2^\circ)(1 + \tan 3^\circ) \cdots (1 + \tan 45^\circ) = \underline{2^{23}}$.

$$(1 + \tan 1^\circ)(1 + \tan 44^\circ) = 1 + \tan 1^\circ + \tan 44^\circ + \tan 1^\circ \cdot \tan 44^\circ$$

$$\because \tan 45^\circ = \tan(1^\circ + 44^\circ) = \frac{\tan 1^\circ + \tan 44^\circ}{1 - \tan 1^\circ \cdot \tan 44^\circ} = 1 \quad \therefore \tan 1^\circ + \tan 44^\circ = 1 - \tan 1^\circ \cdot \tan 44^\circ$$

$$\therefore \tan 1^\circ + \tan 44^\circ + \tan 1^\circ \cdot \tan 44^\circ = 1 \quad \therefore (1 + \tan 1^\circ)(1 + \tan 44^\circ) = 2$$

$$\text{同理, } (1 + \tan 2^\circ)(1 + \tan 43^\circ) = 2, \quad (1 + \tan 3^\circ)(1 + \tan 42^\circ) = 2, \quad \dots$$

$$(1 + \tan 22^\circ)(1 + \tan 23^\circ) = 2.$$

$$\therefore \text{原式} = \underbrace{2 \times 2 \times 2 \times \cdots \times 2}_{24} = 2^{23}$$