



1. 一次分式齐次式:

$$\frac{A \cos x + B \sin x}{C \cos x + D \sin x} = \frac{A + B \tan x}{C + D \tan x}$$

2. 二次齐次式

$$\begin{aligned} A \cos^2 x + B \sin x \cos x + C \sin^2 x &= \frac{A \cos^2 x + B \sin x \cos x + C \sin^2 x}{\cos^2 x + \sin^2 x} \quad (|\div| \text{ 同 } \cos^2 x) \\ &= \frac{A + B \tan x + C \tan^2 x}{1 + \tan^2 x} \end{aligned}$$

【例 1】已知 $\tan \alpha = 3$, 则 $\frac{\sin \alpha - \cos \alpha}{\sin \alpha + 2 \cos \alpha} = \underline{\quad}$.

$$\frac{(\sin \alpha - \cos \alpha) \div \cos \alpha}{(\sin \alpha + 2 \cos \alpha) \div \cos \alpha} = \frac{\tan \alpha - 1}{\tan \alpha + 2} = \frac{3-1}{3+2} = \frac{2}{5}$$

【例 2】已知 $\tan \alpha = 2$ ，则 $1 + \sin \alpha \cos \alpha = \underline{\frac{7}{5}}$ ， $\sin^2 \alpha - 2 \sin \alpha \cos \alpha = \underline{0}$ 。

$$(1) \cos^2 \alpha + \sin^2 \alpha + \sin \alpha \cos \alpha = \frac{\tan^2 \alpha + 1 + \tan \alpha + 1}{1 + \tan^2 \alpha} = \frac{4 + 2 + 1}{5} \quad (2) \sin^2 \alpha - 2 \sin \alpha \cos \alpha = \frac{\sin^2 \alpha - 2 \cos \alpha \sin \alpha}{\sin^2 \alpha + \cos^2 \alpha} = \frac{1 \tan^2 \alpha - 2 \tan \alpha}{1 + \tan^2 \alpha} = \frac{4 - 4}{1 + 4} = 0$$

【例 3】已知 $\alpha \in \mathbb{R}$ ，且 $\log_4(2 \sin \alpha + \cos \alpha) + \log_4(\sin \alpha + 2 \cos \alpha) = 1$ ，则 $\tan \alpha =$ (C)

(A) $\frac{1}{2}$ (B) 2 (C) $\frac{1}{2}$ 或 2 (D) 不存在

$$\log_4(2 \sin \alpha + \cos \alpha) + \log_4(\sin \alpha + 2 \cos \alpha) = \log_4[(2 \sin \alpha + \cos \alpha)(\sin \alpha + 2 \cos \alpha)] = \log_4(5 \sin \alpha \cos \alpha + 2 \cos^2 \alpha + 2 \sin^2 \alpha) = \log_4\left(\frac{2 + 2 \tan^2 \alpha + 1 + 2 \tan \alpha}{1 + \tan^2 \alpha}\right) = 1$$

$$\frac{2 + 2 \tan^2 \alpha + 1 + 2 \tan \alpha}{1 + \tan^2 \alpha} = 4$$

$$2 + 2 \tan^2 \alpha + 5 \tan \alpha = 4 + 4 \tan^2 \alpha$$

$$2 \tan^2 \alpha - 5 \tan \alpha + 2 = 0$$

$$(2 \tan \alpha - 1)(\tan \alpha - 2) = 0$$

【例 4】已知 $\sin^3 \theta - \sin \theta \cos \theta - 2 \cos^3 \theta = 0$ ， $\angle \theta$ 为锐角，则 $\frac{\sin \theta + \cos \theta}{\sin \theta} + \cos^2 \theta =$ _____。

$$\frac{\cos^2 \theta}{\sin^2 \theta \cos \theta} = \frac{1}{1 + \tan^2 \theta} = \frac{1}{5}$$

$$\frac{\tan^2 \alpha - \tan \alpha - 2}{1 + \tan^2 \alpha} = 0$$

$$\frac{\tan \alpha + 1}{\tan \alpha} + \cos^2 \theta$$

$$= \frac{3}{2} + \frac{1}{5} = \frac{15}{10} + \frac{2}{10} = \frac{17}{10}$$

$$(\tan \alpha - 2)(\tan \alpha + 1) = 0$$

$$\tan \alpha = 2 \text{ 或 } -1 \left(\frac{3}{2}\right)$$

