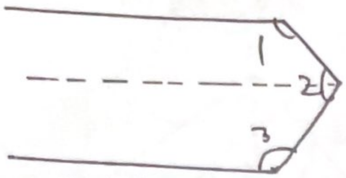
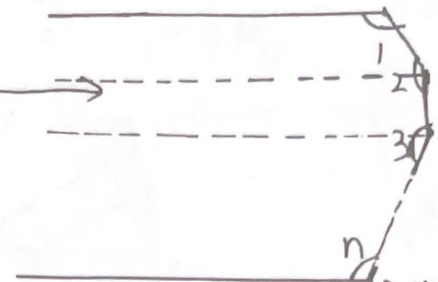
几何模型①

一.平行模型

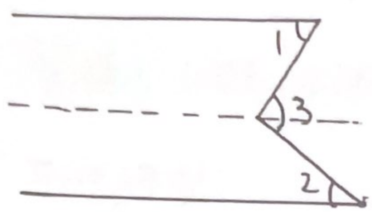


$\angle 1+\angle 2+\angle 3=360^\circ$

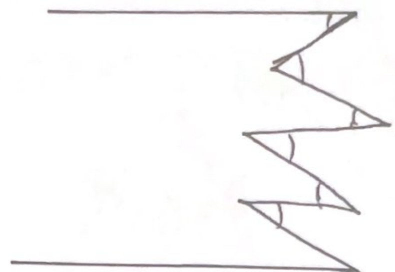


作平行 ⇒ 存在多少层就有多少180°

$\angle_1+\angle_2+\cdots+\angle_n=(n-1)\times 180^\circ$

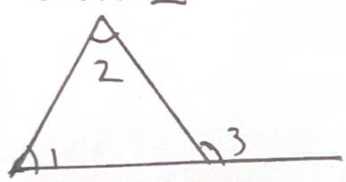


$\angle_1+\angle_2=\angle 3$

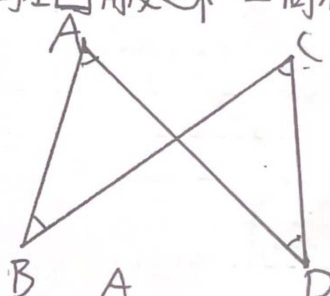


向左凸角度之和=向右凸角度之和

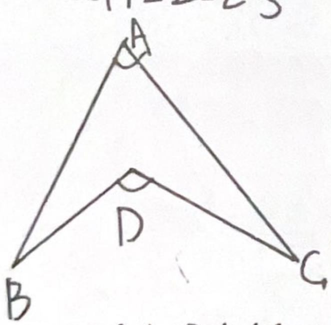
二.导角模型



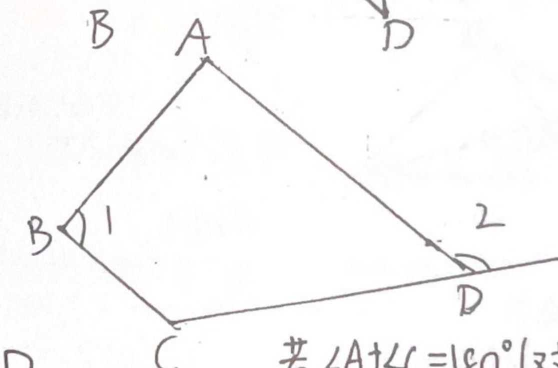
$\angle 1+\angle 2=\angle 3$



$\angle A+\angle B=\angle C+\angle D$



$\angle A+\angle B+\angle C=\angle D$



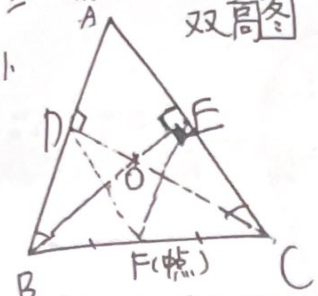
若$\angle A+\angle C=180^\circ$(对角)

则$\angle 1=\angle_2$

几何模型②

三.三角形模型

1. 双高图



① $AB\cdot CD=AC\cdot BE$(等面积)

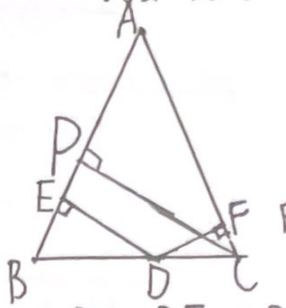
② $AD\cdot AB=AE\cdot AC$(相似)

③ $OD\cdot OC=OE\cdot OB$

④ B.C.E,D四点共圆

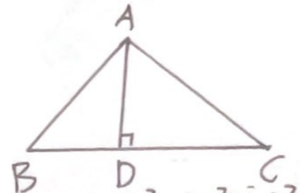
⑤ $DF=EF=BF=CF$

2. 等腰三角形



$DE+DF=CP$

公共高模型



$\begin{cases}AB^2-BD^2=AC^2-CD^2\\AB^2-AC^2=BD^2-CD^2\end{cases}$

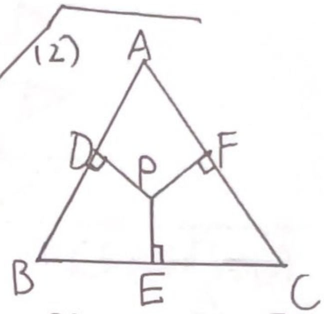
3.等边三角形模型

(1) $S_{等边}=\frac{\sqrt{3}}{4}a^2$


$S_{60-120菱形}=\frac{\sqrt{3}}{4}a^2\times2$

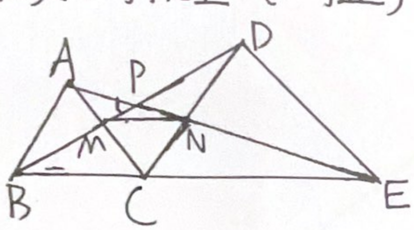

$S_{正六边形}=\frac{\sqrt{3}}{4}a^2\times6$

(2)



$PD+PE+PF=\frac{\sqrt{3}}{2}a$ (a为边长)

(3)手拉手模型(三等边)



① 三组全等 $\begin{cases}\triangle ACE\cong\triangle BCD\\\triangle CNE\cong\triangle CMD\\\triangle CMB\cong\triangle CNA\end{cases}$

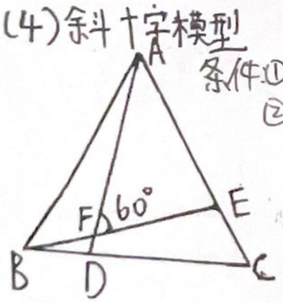
② 三个等边△: $\triangle ABC,\triangle DCE,\triangle CMN$

③ $MN\parallel BE$

④ AE, BD夹角为60°

⑤ PC平分∠BPE,不平分∠ACD

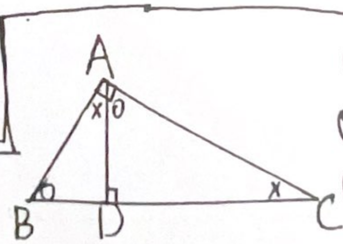
(4)斜十字模型



条件: ① $BD=CE$ ② △ABC等边

结论: ① $\triangle ABD\cong\triangle BCE$ ② $\angle AFE=60°$

4.直角三角形



①角度相等

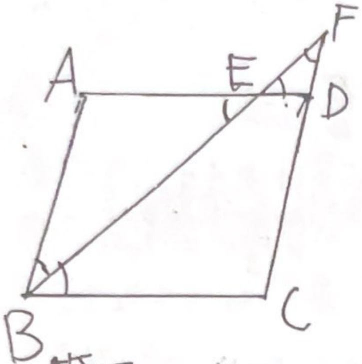
②斜高公式: $AD=\frac{AB\cdot AC}{BC}$

③射影定理

$\begin{cases}AB^2=BD\cdot BC\\AD^2=BD\cdot DC\\AC^2=CD\cdot BC\end{cases}$

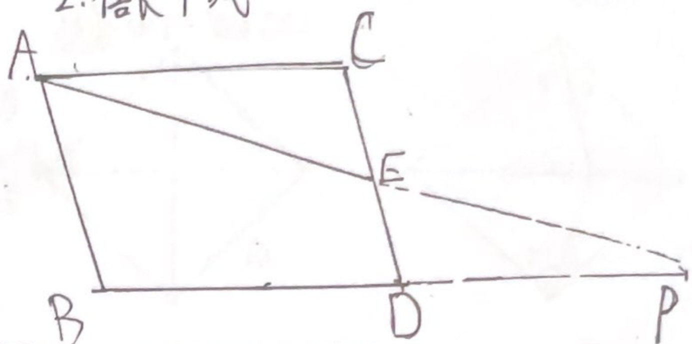
几何模型③

1. 平行四边形+角平分线 ⟶ 等腰△



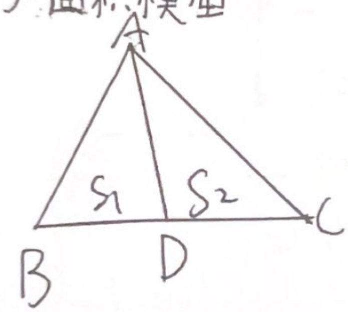
等腰△：△ABE，△DEF，△BCF

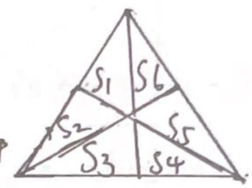
2. 倍长中线



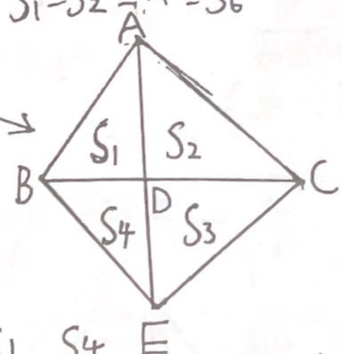
△ACE ≌ △PDE

3. 面积模型



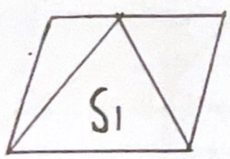


$S_1=S_2=\cdots=S_6$

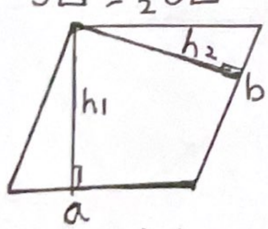


$$\frac{S_1}{S_2}=\frac{BD}{CD}$$

$$\frac{S_1}{S_2}=\frac{S_4}{S_3}\quad S_1\cdot S_3=S_2\cdot S_4$$

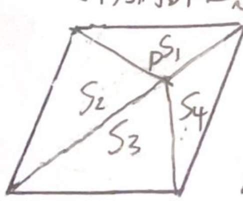


$S_{\triangle}=\frac{1}{2}S_{\square}$

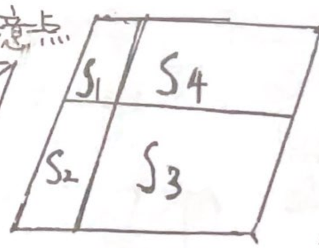


$a\cdot h_1=b\cdot h_2$

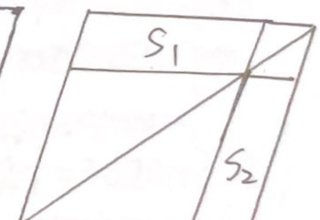
(P为内部任意点



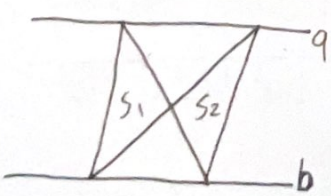
$S_1+S_3=S_2+S_4$



$S_1\cdot S_3=S_2\cdot S_4$



$S_1=S_2$



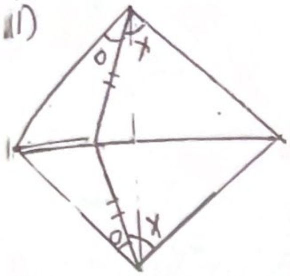
若 $a\parallel b$ 则 $S_1=S_2$

几何模型④

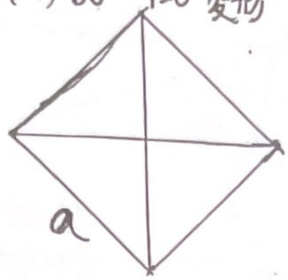
特殊平行四边形模型

1.菱形

(1)



(2) 60°-120°菱形

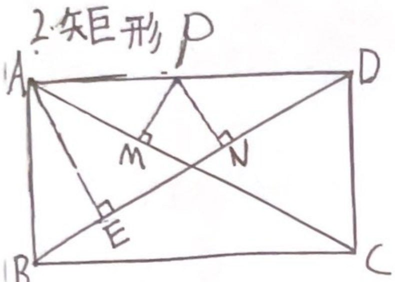


① 面积 $\frac{\sqrt{3}}{4}a^2\times2$

② 边长 : 短对角线 : 长对角线

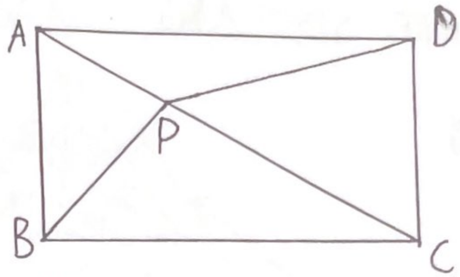
$1:1:\sqrt{3}$

2.矩形



$PM+PN=AE=\frac{长\times宽}{对角线}$

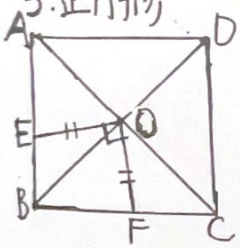
(P为矩形边上任一点)



$PA^2+PC^2=PB^2+PD^2$

(P为平面内任意一点)

3.正方形

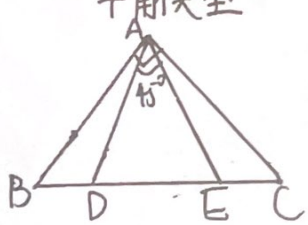


1. $OE=OF \rightleftharpoons OE\perp OF$

② $\begin{cases}\triangle OEB\cong\triangle OFC\\ \triangle OAE\cong\triangle OBF\end{cases}$

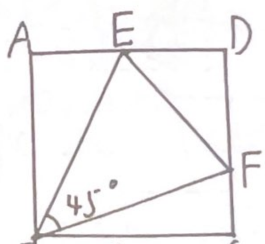
③ $S_{四OEBF}=\frac{1}{4}S_{正}$

半角模型



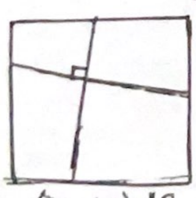
等腰Rt△

$BD^2+CE^2=DE^2$

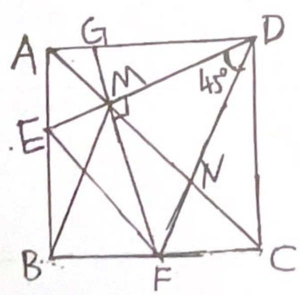


正方形

$AE+CF=EF$



斜十字型



条件: $MB=MD=MF$

结论:

① $MB=MD$

② 等腰Rt△DMF

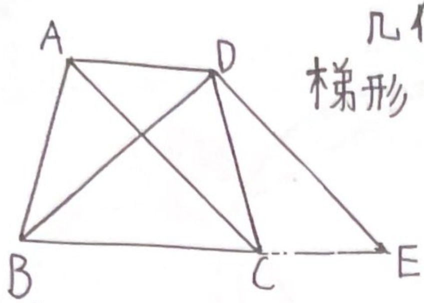
③ 半角模型

$AM^2+CN^2=MN^2$

④ $DE=GF$ (斜十字)

$AE+CF=BF$

几何模型⑤



梯形

当对角线垂直或相等时，平移对角线

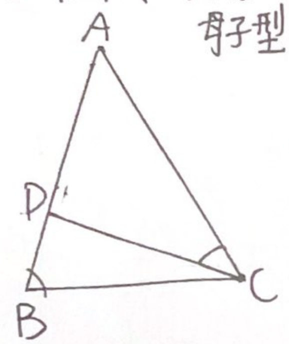
①构造直角或等腰△

②集中对角线和上下底

③面积转化：$S_{梯ABCD}=S_{\triangle BDE}$

5. 相似三角形

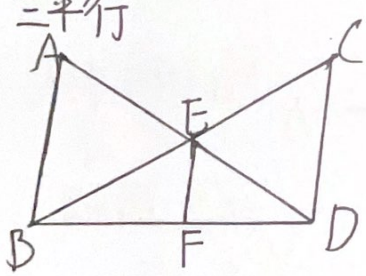
母子型



当$\angle ACD=\angle B$时，$\triangle ABC\backsim\triangle AED$

∴ $AC^2=AD\cdot AB$

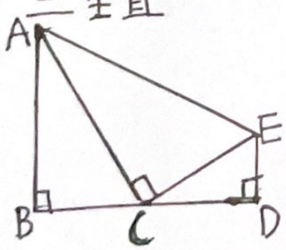
三平行



(1) $\frac{1}{AB}+\frac{1}{CD}=\frac{1}{EF}$

(2) $\frac{1}{S_{\triangle ABD}}+\frac{1}{S_{\triangle BCD}}=\frac{1}{S_{\triangle BDE}}$

三垂直



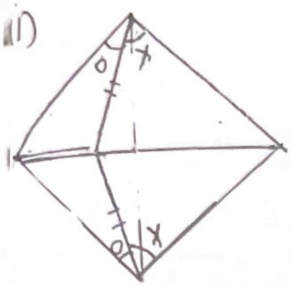
∴ $AB\cdot DE=BC\cdot CD$

几何模型④

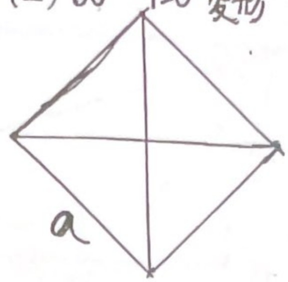
特殊平行四边形模型

1.菱形

(1)



(2) 60°-120°菱形

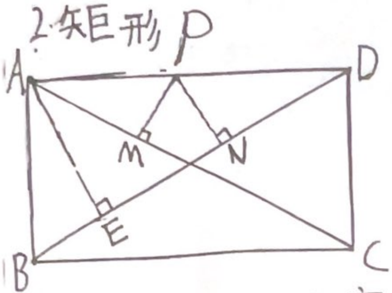


① 面积 $\frac{\sqrt{3}}{4}a^2 \times 2$

② 边长 : 短对角线 : 长对角线

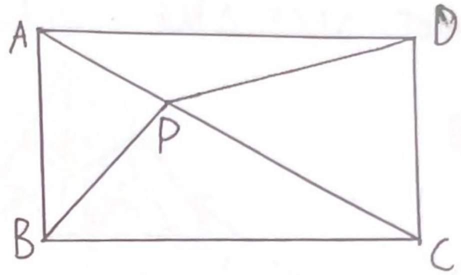
$1 : 1 : \sqrt{3}$

2.矩形



$PM+PN=AE=\frac{长\times宽}{对角线}$

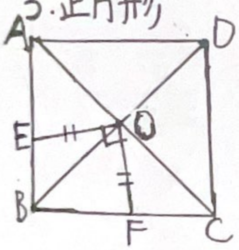
(P为矩形边上任一点)



$PA^2+PC^2=PB^2+PD^2$

(P为平面内任意一点)

3.正方形



1. $OE=OF \rightleftharpoons OE\perp OF$

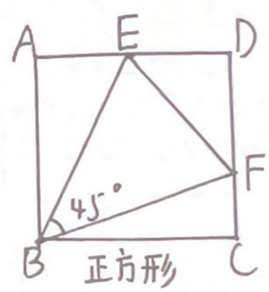
② $\begin{cases}\triangle OEB\cong\triangle OFC\\ \triangle OAE\cong\triangle OBF\end{cases}$

③ $S_{四OEBF}=\frac{1}{4}S_{正}$

半角模型

等腰Rt△

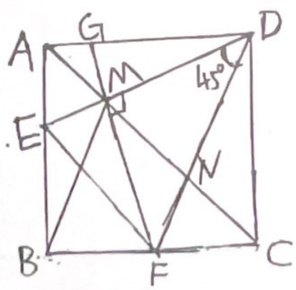
$BD^2+CE^2=DE^2$



正方形

$AE+CF=EF$

斜十字架



条件: $MB=MD=MF$

结论:

① $MB=MD$

② 等腰Rt△DMF

③ 半角模型

$AM^2+CN^2=MN^2$

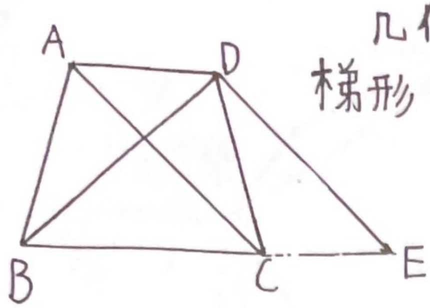
④ $DE=GF$

(斜十字)

$AE+CF=BF$

几何模型⑤

梯形



当对角线垂直或相等时，平移对角线

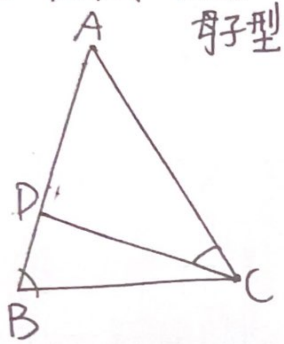
①构造直角或等腰△

②集中对角线和上下底

③面积转化：$S_{梯ABCD}=S_{\triangle BDE}$

5. 相似三角形

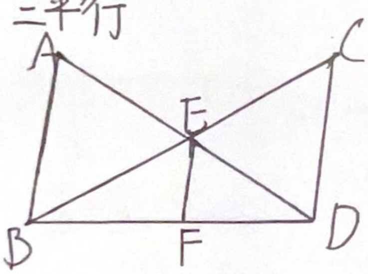
母子型



当$\angle ACD=\angle B$时，$\triangle ABC\backsim\triangle AED$

∴ $AC^2=AD\cdot AB$

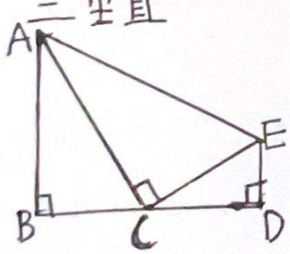
三平行



(1) $\frac{1}{AB}+\frac{1}{CD}=\frac{1}{EF}$

(2) $\frac{1}{S_{\triangle ABD}}+\frac{1}{S_{\triangle BCD}}=\frac{1}{S_{\triangle BDE}}$

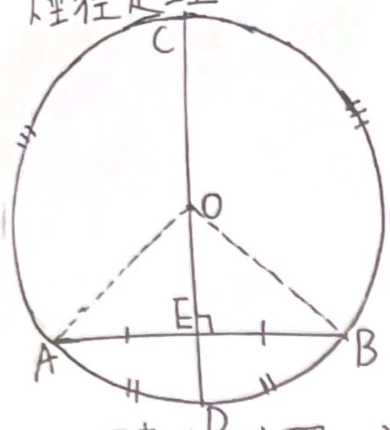
三垂直



∴ $AB\cdot DE=BC\cdot CD$

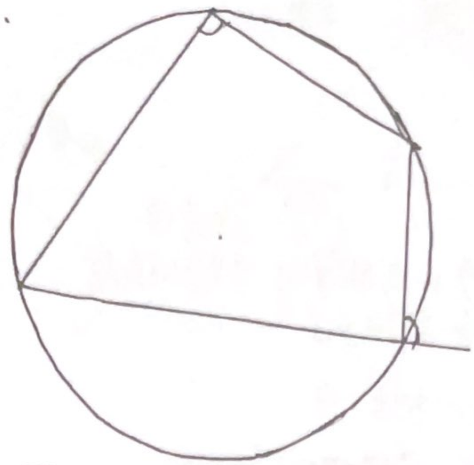
几何模型⑥

1.垂径定理



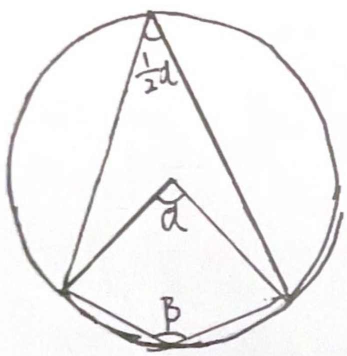
①直径(过圆心)
②$CD \perp AB$
③$AE=BF$
④$\overset{\frown}{AD}=\overset{\frown}{BD}$
⑤$\overset{\frown}{AC}=\overset{\frown}{BC}$

2.



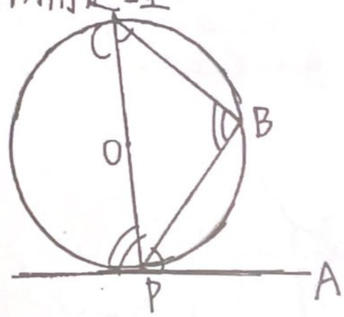
圆内接四边形

①对角互补
②外角=内对角



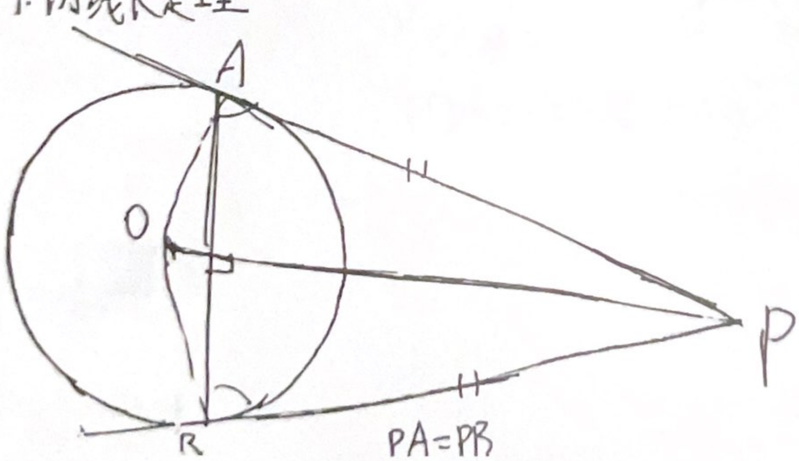
$\frac{1}{2}\alpha+\beta=180°$

3.弦切角定理



$\angle BPA=\angle BCP$

4.切线长定理

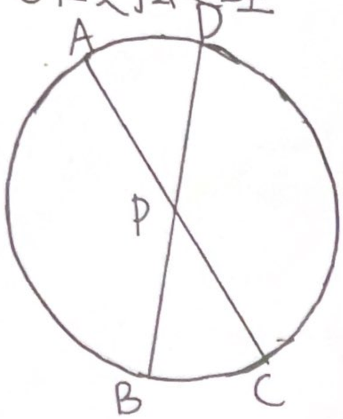


$PA=PB$

几何模型⑦

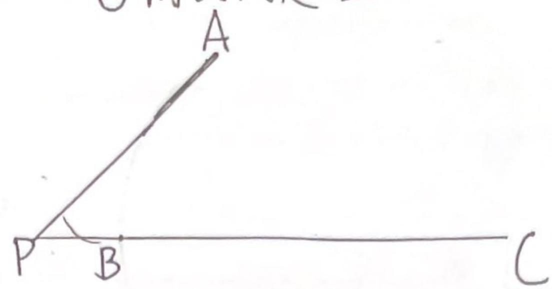
圆幂定理

①相交弦定理



$PA \cdot PC = PB \cdot PD$

②切割线定理

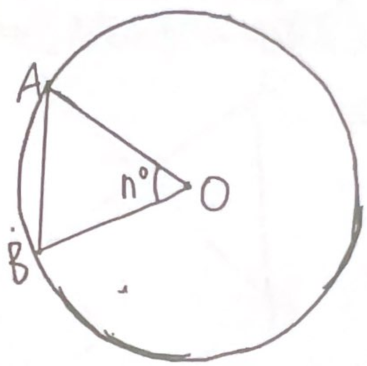


$PA^2 = PB \cdot PC$

6.圆的相关计算

$L_{弧长} = \frac{n^\circ}{360^\circ} \cdot 2\pi r$

$S_{扇} = \frac{n^\circ}{360^\circ} \cdot \pi r^2$



$S_{弓} = S_{扇} - S_{\triangle}$

$S_{侧} = \pi \cdot r \cdot l$

↓

展开图:扇形

$V_{体} = \frac{1}{3}\pi r^2 h$