

60-02

2000 AMC10 试题

Problem 1

In the year 2001, the United States will host the International Mathematical Olympiad. Let I , M , and O be distinct positive integers such that the product $I \cdot M \cdot O = 2001$. What is the largest possible value of the sum $I + M + O$?

在 2001 年，美国将举行国际数学奥林匹克。令 I, M, O 为不同的正整数，满足乘积 $I \cdot M \cdot O = 2001$ 。则 $I + M + O$ 的最大可能值是多少？

- (A) 23 (B) 55 (C) 99 (D) 111 (E) 671

Problem 2

$$2000(2000^{2000}) =$$

- (A) 2000^{2001} (B) 4000^{2000} (C) 2000^{4000} (D) $4,000,000^{2000}$ (E) $2000^{4,000,000}$

Problem 3

Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?

每天，Jenny 都会吃掉那天开始时罐子里剩余果冻豆的 20%。在第二天结束时，还剩 32 颗。问一开始罐子里有多少颗果冻豆？

- (A) 40 (B) 50 (C) 55 (D) 60 (E) 75

Problem 4

Chandra pays an on-line service provider a fixed monthly fee plus an hourly charge for connect time. Her December bill was \$12.48, but in January her bill was \$17.54 because she used twice as much connect time as in December. What is the fixed monthly fee?

Chandra 向在线服务提供商支付固定的每月费用，外加连接时间的按小时计算的费用。她 12 月份的账单是 12.48 美元，但 1 月份的账单是 17.54 美元因为她使用的连接时间是 12 月份的两倍。问固定月费是多少？

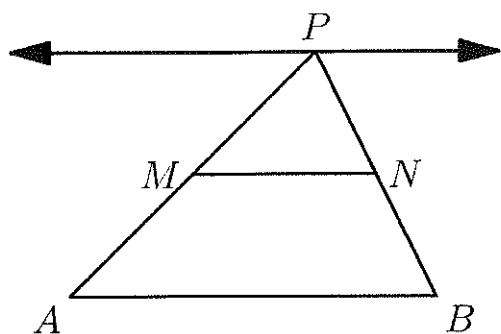
- (A) \$2.53 (B) \$5.06 (C) \$6.24 (D) \$7.42 (E) \$8.77

Problem 5

Points M and N are the midpoints of sides PA and PB of $\triangle PAB$. As P moves along a line that is parallel to side AB , how many of the four quantities listed below change?

点 M 和 N 是 $\triangle PAB$ 的边 PA 和 PB 的中点。当 P 沿着平行于边 AB 的直线移动时，下面列出的 4 个量中哪几个会发生改变？

- (a) the length of the segment MN | 线段 MN 的长度
- (b) the perimeter of $\triangle PAB$ | $\triangle PAB$ 的周长
- (c) the area of $\triangle PAB$ | $\triangle PAB$ 的面积
- (d) the area of trapezoid $ABNM$ | 梯形 $ABNM$ 的面积



- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Problem 6

The Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, 21, \dots$ starts with two 1s, and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

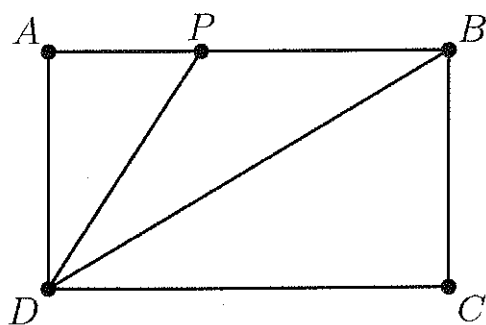
斐波那契数列 $1, 1, 2, 3, 5, 8, 13, 21, \dots$ 以两个 1 开始, 之后的每一项是它的前两项之和。问 0 到 9 这 10 个数字中, 哪个数字最后一个出现在斐波那契数列各项的个位上?

- (A) 0 (B) 4 (C) 6 (D) 7 (E) 9

Problem 7

In rectangle $ABCD$, $AD = 1$, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?

在矩形 $ABCD$ 中, $AD = 1$, P 在 $\overline{AB}$ 上, 且 $\overline{DB}$ 和 $\overline{DP}$ 三等分 $\angle ADC$ 。那么 $\triangle BDP$ 的周长是多少?



- (A) $3 + \frac{\sqrt{3}}{3}$ (B) $2 + \frac{4\sqrt{3}}{3}$ (C) $2 + 2\sqrt{2}$ (D) $\frac{3 + 3\sqrt{5}}{2}$ (E) $2 + \frac{5\sqrt{3}}{3}$

Problem 8

At Olympic High School, $\frac{2}{5}$ of the freshmen and $\frac{4}{5}$ of the sophomores took the AMC 10. Given that the number of freshmen and sophomore contestants was the same, which of the following must be true?

在奥林匹克高中， $\frac{2}{5}$ 的高一学生和 $\frac{4}{5}$ 的高二学生参加了 AMC10 考试。已知高一和高二参赛者的数量相同，以下哪项一定是正确的？

(A) There are five times as many sophomores as freshmen. | 高二学生总数是高一学生总数的 5 倍。

(B) There are twice as many sophomores as freshmen. | 高二学生总数是高一学生总数的 2 倍。

(C) There are as many freshmen as sophomores. | 高二学生总数和高一学生总数相同。

(D) There are twice as many freshmen as sophomores. | 高一学生总数是高二学生总数的 2 倍。

(E) There are five times as many freshmen as sophomores. | 高一学生总数是高二学生总数的 5 倍。

Problem 9

If $|x - 2| = p$, where $x < 2$, then $x - p =$

若 $|x - 2| = p$, 其中 $x < 2$, 那么 $x - p =$

(A) -2 (B) 2 (C) $2 - 2p$ (D) $2p - 2$ (E) $|2p - 2|$

Problem 10

The sides of a triangle with positive area have lengths 4, 6, and x . The sides of a second triangle with positive area have lengths 4, 6, and y . What is the smallest positive number that is **not** a possible value of $|x - y|$?

面积为正的一个三角形的三边长分别为 4, 6 和 x . 第二个面积为正的三角形的三边长为 4, 6 和 y . 则不可能是 $|x - y|$ 的值中, 最小的正数是多少?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

Problem 11

Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

从 4 和 18 之间选择 2 个不同的质数。当将它们的乘积减去它们的和，会得到下面哪个数？

- (A) 22 (B) 60 (C) 119 (D) 180 (E) 231

Problem 12

Figures 0, 1, 2, and 3 consist of 1, 5, 13, and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100?

图 0, 1, 2, 和 3 分别有 1 个, 5 个, 13 个和 25 个不重叠的单位正方形组成。如果这样的规律继续下去, 那么在图 100 中将有多少个不重叠的单位正方形?



Figure
0



Figure
1

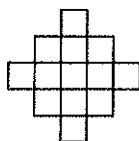


Figure
2

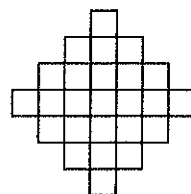


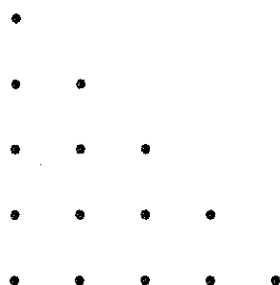
Figure
3

- (A) 10401 (B) 19801 (C) 20201 (D) 39801 (E) 40801

Problem 13

There are 5 yellow pegs, 4 red pegs, 3 green pegs, 2 blue pegs, and 1 orange peg to be placed on a triangular peg board. In how many ways can the pegs be placed so that no (horizontal) row or (vertical) column contains two pegs of the same color?

三角形钉板上有 5 个黄色钉、4 个红色钉、3 个绿色钉、2 个蓝色钉和 1 个橙色钉。有多少种放置钉子的方式, 使得 (水平) 行或 (垂直) 列中没有两个相同颜色的钉子?



- (A) 0 (B) 1 (C) $5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!$ (D) $\frac{15!}{5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!}$ (E) $15!$

Problem 14

Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were 71, 76, 80, 82, and 91. What was the last score Mrs. Walter entered?

Walter 太太给一个班 5 个学生做了次数学测试。她将学生的考试分数以随机顺序输入一张电子表格,在输入每个分数后,电子表格会重新计算班级平均分。Walter 太太注意到,每次输入分数后,平均值总是一个整数。得分(按升序排列)为 71, 76, 80, 82 和 91。则 Walter 太太最后输入的分数是多少?

- (A) 71 (B) 76 (C) 80 (D) 82 (E) 91

Problem 15

Two non-zero real numbers, a and b , satisfy $ab = a - b$. Find a possible value of $\frac{a}{b} + \frac{b}{a} - ab$.

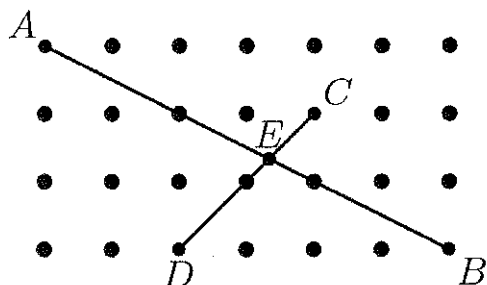
两个非零实数 a 和 b 满足 $ab = a - b$, 求 $\frac{a}{b} + \frac{b}{a} - ab$ 的一个可能值。

- (A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 2

Problem 16

The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segment AB meets segment CD at E . Find the length of segment AE .

下图显示了 28 个格点，每个格点和它最近的邻居的距离为 1 个单位。线段 AB 交线段 CD 于 E 。求线段 AE 的长度。



- (A) $\frac{4\sqrt{5}}{3}$ (B) $\frac{5\sqrt{5}}{3}$ (C) $\frac{12\sqrt{5}}{7}$ (D) $2\sqrt{5}$ (E) $\frac{5\sqrt{65}}{9}$

Problem 17

Boris has an incredible coin changing machine. When he puts in a quarter, it returns five nickels; when he puts in a nickel, it returns five pennies; and when he puts in a penny, it returns five quarters. Boris starts with just one penny. Which of the following amounts could Boris have after using the machine repeatedly?

Boris 有一台不可思议的硬币兑换机。当他放入一枚 25 美分硬币时，它会返回五枚 5 分镍币；当他放入一枚五分镍币时，它会返回五枚 1 分硬币；当他投进一枚 1 分硬币时，它会返回五枚 25 美分硬币。Boris 从一枚 1 分硬币开始，在他反复使用机器后可能会得到以下哪个金额？

- (A) \$3.63 (B) \$5.13 (C) \$6.30 (D) \$7.45 (E) \$9.07

Problem 18

Charlyn walks completely around the boundary of a square whose sides are each 5 km long. From any point on her path she can see exactly 1 km horizontally in all directions. What is the area of the region consisting of all points Charlyn can see during her walk, expressed in square kilometers and rounded to the nearest whole number?

Charlyn 完全绕着一个各边长为 5 公里的正方形的边界行走。从她路径上的任何一点,她都可以看到各个方向上水平距离 1 公里内的地方。Charlyn 步行时能看到的所有点组成的区域面积是多少平方公里?结果四舍五入到最接近的整数。

- (A) 24 (B) 27 (C) 39 (D) 40 (E) 42

Problem 19

Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is m times the area of the square. The ratio of the area of the other small right triangle to the area of the square is

过一个直角三角形斜边上一点,分别作两条直角边的平行线,将三角形分成一个正方形以及两个小的直角三角形。其中一个小的直角三角形的面积是正方形面积的 m 倍。那么另一个小的直角三角形的面积与正方形的面积之比是

- (A) $\frac{1}{2m+1}$ (B) m (C) $1-m$ (D) $\frac{1}{4m}$ (E) $\frac{1}{8m^2}$

Problem 20

Let A , M , and C be nonnegative integers such that $A + M + C = 10$. What is the maximum value of $A \cdot M \cdot C + A \cdot M + M \cdot C + C \cdot A$?

令 A, M, C 为非负整数, 满足 $A + M + C = 10$ 。那么

$A \cdot M \cdot C + A \cdot M + M \cdot C + C \cdot A$ 的最大值是多少?

- (A) 49 (B) 59 (C) 69 (D) 79 (E) 89

Problem 21

If all alligators are ferocious creatures and some creepy crawlers are alligators, which statement(s) must be true?

- I. All alligators are creepy crawlers.
- II. Some ferocious creatures are creepy crawlers.
- III. Some alligators are not creepy crawlers.

- (A) I only (B) II only (C) III only (D) II and III only (E) None must be true

如果所有的短吻鳄都是凶猛的生物, 而一些令人毛骨悚然的爬行动物是短吻鳄,

那么哪种说法是正确的?

I. 所有的短吻鳄都是令人毛骨悚然的爬行动物。

II. 一些凶猛的生物是令人毛骨悚然的爬行动物。

III. 一些短吻鳄不是令人毛骨悚然的爬行动物。

- (A) 只有 I (B) 只有 II (C) 只有 III (D) 只有 II 和 III (E) 没有一个正确的

Problem 22

One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never

zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

一天早上, Angela 家的每个成员都喝了一杯 8 盎司的牛奶咖啡。每杯咖啡和牛奶的量因杯而异, 但从不为零。Angela 喝了牛奶总量的四分之一和咖啡总量的六分之一。问家里有多少人?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Problem 23

When the mean, median, and mode of the list $10, 2, 5, 2, 4, 2, x$ are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x ?

当将一列数 $10, 2, 5, 2, 4, 2, x$ 的平均数, 中位数和众数从小到大排序, 它们形成一个等差数列 (不是常数数列)。那么 x 的所有可能的实数值之和是多少?

- (A) 3 (B) 6 (C) 9 (D) 17 (E) 20

Problem 24

Let f be a function for which $f\left(\frac{x}{3}\right) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

令 f 为满足 $f\left(\frac{x}{3}\right) = x^2 + x + 1$ 的函数。求使得 $f(3z) = 7$ 的所有 z 值之和。

- (A) $-\frac{1}{3}$ (B) $-\frac{1}{9}$ (C) 0 (D) $\frac{5}{9}$ (E) $\frac{5}{3}$

Problem 25

In year N , the 300th day of the year is a Tuesday. In year $N + 1$, the 200th day is also a Tuesday. On what day of the week did the 100th day of year $N - 1$ occur?

- (A) Thursday (B) Friday (C) Saturday (D) Sunday (E) Monday

在年份为 N 的这一年，第 300 天是个周二。在年份为 $N+1$ 的这一年，第 200 天也是个周二。那么在年份为 $N-1$ 的这一年，第 100 天是周几？

- (A) 周四 (B) 周五 (C) 周六 (D) 周日 (E) 周一