

## 2002 AMC10A 试题

### Problem 1

The ratio  $\frac{10^{2000} + 10^{2002}}{10^{2001} + 10^{2001}}$  is closest to which of the following numbers?

比值  $\frac{10^{2000} + 10^{2002}}{10^{2001} + 10^{2001}}$  最接近下面哪个数?

- (A) 0.1    (B) 0.2    (C) 1    (D) 5    (E) 10

### Problem 2

Given that  $a, b$ , and  $c$  are non-zero real numbers, define  $(a, b, c) = \frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ .

Find  $(2, 12, 9)$ .

已知  $a, b$ , 和  $c$  都是非零实数, 定义  $(a, b, c) = \frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ , 求  $(2, 12, 9)$ 。

- (A) 4    (B) 5    (C) 6    (D) 7    (E) 8

### Problem 3

According to the standard convention for exponentiation,

$$2^{2^{2^2}} = 2^{\left(2^{\left(2^2\right)}\right)} = 2^{16} = 65,536.$$

If the order in which the exponentiations are performed is changed, how many other values are possible?

根据指数的标准惯例, 我们有

$$2^{2^{2^2}} = 2^{\left(2^{\left(2^2\right)}\right)} = 2^{16} = 65,536.$$

如果指数的运算次序发生改变, 那么我们还可以得到多少个其他的值?

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

#### Problem 4

For how many positive integers  $m$  is there at least 1 positive integer  $n$  such that  $mn \leq m + n$ ?

正整数  $m$  有多少个值, 使得至少存在一个正整数  $n$ , 满足  $mn \leq m + n$ ?

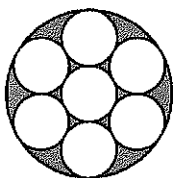
- (A) 4      (B) 6      (C) 9      (D) 12      (E) 无穷多个

#### Problem 5

Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.

图中的每个小圆的半径均为 1。最里面的圆和它周围的 6 个圆都相切, 且这 6 个

圆都和大圆相切, 并和相邻的圆也相切。求阴影部分的面积。



- (A)  $\pi$       (B)  $1.5\pi$       (C)  $2\pi$       (D)  $3\pi$       (E)  $3.5\pi$

#### Problem 6

From a starting number, Cindy was supposed to subtract 3, and then divide by 9, but instead, Cindy subtracted 9, then divided by 3, getting 43. If the correct instructions were followed, what would the result be?

从一个初始数开始, Cindy 本应该先减去 3, 然后除以 9, 但是 Cindy 先减去了 9,

然后除以 3, 结果得到 43。如果遵循正确的步骤, 那么结果应该是多少?

- (A) 15      (B) 34      (C) 43      (D) 51      (E) 138

#### Problem 7

A  $45^\circ$  arc of circle A is equal in length to a  $30^\circ$  arc of circle B. What is the ratio of circle A's area and circle B's area?

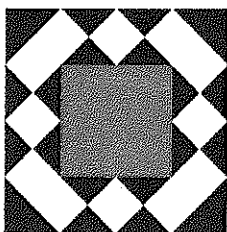
圆 A 的一段  $45^\circ$  圆弧和圆 B 的一段  $30^\circ$  圆弧等长。那么圆 A 的面积和圆 B 的面积之比是多少?

- (A)  $4/9$       (B)  $2/3$       (C)  $5/6$       (D)  $3/2$       (E)  $9/4$

### Problem 8

Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let  $B$  be the total area of the blue triangles,  $W$  the total area of the white squares, and  $R$  the area of the red square. Which of the following is correct?

Besty 设计了一面旗帜，由若干蓝色三角形，若干白色正方形以及一个位于中心的红色正方形组成，如图所示。令  $B$  表示蓝色三角形的总面积， $W$  表示白色正方形的总面积， $R$  表示红色正方形的面积。则下面哪个是正确的？



- (A)  $B = W$       (B)  $W = R$       (C)  $B = R$       (D)  $3B = 2R$       (E)  $2R = W$

### Problem 9

There are 3 numbers  $A$ ,  $B$ , and  $C$ , such that  $1001C - 2002A = 4004$ , and  $1001B + 3003A = 5005$ . What is the average of  $A$ ,  $B$ , and  $C$ ?

有 3 个数， $A, B, C$ ，满足  $1001C - 2002A = 4004$ ，且  $1001B + 3003A = 5005$ 。

那么  $A, B$ ，和  $C$  的平均值是多少？

- (A) 1      (B) 3      (C) 6      (D) 9      (E) 不能唯一确定

### Problem 10

What is the sum of all of the roots of  $(2x + 3)(x - 4) + (2x + 3)(x - 6) = 0$ ?

方程  $(2x + 3)(x - 4) + (2x + 3)(x - 6) = 0$  的所有根之和是多少？

- (A)  $7/2$       (B) 4      (C) 5      (D) 7      (E) 13

**Problem 11**

Jamal wants to save 30 files onto disks, each with 1.44 MB space. 3 of the files take up 0.8 MB each, 12 of the files take up 0.7 MB each, and the rest take up 0.4 MB each. It is not possible to split a file onto 2 different disks. What is the smallest number of disks needed to store all 30 files?

Jamal 希望将 30 个文件保存到磁盘上, 每个磁盘都有 1.44 MB 的空间。其中 3 个文件每个占用 0.8 MB, 12 个文件每个占用 0.7 MB, 其余文件每个占用 0.4 MB。无法将同一个文件拆分到两个不同的磁盘上。存储全部 30 个文件所需的最少磁盘数是多少?

- (A) 12      (B) 13      (C) 14      (D) 15      (E) 16

**Problem 12**

Mr. Earl E. Bird leaves home every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

Earl E. Bird 先生每天早上 8 点离开家去上班。如果他以每小时 40 英里的平均速度开车, 他将迟到 3 分钟。如果他以每小时 60 英里的平均速度开车, 他将提前 3 分钟到达。Earl E. Bird 需要以每小时多少英里的速度开车才能保证准时上班?

- (A) 45      (B) 48      (C) 50      (D) 55      (E) 58

**Problem 13**

Given a triangle with side lengths 15, 20, and 25, find the triangle's smallest altitude.

一个三角形的三边长为 15, 20, 和 25, 求三角形最短的高。

- (A) 6      (B) 12      (C) 12.5      (D) 13      (E) 15

**Problem 14**

Both roots of the quadratic equation  $x^2 - 63x + k = 0$  are prime numbers. The number of possible values of  $k$  is

- (A) 0      (B) 1      (C) 2      (D) 4      (E) more than 4

一元二次方程  $x^2 - 63x + k = 0$  的两个根都是质数。那么  $k$  的可能值的个数为

- (A) 0    (B) 1    (C) 2    (D) 4    (E) 多于 4 个

**Problem 15**

Using the digits 1, 2, 3, 4, 5, 6, 7, and 9, form 4 two-digit prime numbers, using each digit only once. What is the sum of the 4 prime numbers?

使用数字 1, 2, 3, 4, 5, 6, 7, 和 9, 每个数字使用一次, 形成 4 个两位质数。问这 4 个质数的和是多少?

- (A) 150    (B) 160    (C) 170    (D) 180    (E) 190

**Problem 16**

Let  $a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5$ . What is  $a + b + c + d$ ?

令  $a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5$ , 则  $a + b + c + d$  是多少?

- (A)  $-5$     (B)  $-10/3$     (C)  $-7/3$     (D)  $5/3$     (E) 5

**Problem 17**

Sarah pours 4 ounces of coffee into a cup that can hold 8 ounces. Then she pours 4 ounces of cream into a second cup that can also hold 8 ounces. She then pours half of the contents of the first cup into the second cup, completely mixes the contents of the second cup, then pours half of the contents of the second cup back into the first cup. What fraction of the contents in the first cup is cream?

Sarah 将 4 盎司的咖啡倒入一个容积为 8 盎司的杯中。接着她将 4 盎司的奶油倒入第二个容积也为 8 盎司的杯中。然后她将第一个杯中一半的液体倒入第二个杯中, 与第二个杯中的液体均匀混合后, 接着又将第二个杯中液体的一半重新倒回第一个杯中。问现在第一个杯中有多少比例是奶油?

- (A)  $1/4$     (B)  $1/3$     (C)  $3/8$     (D)  $2/5$     (E)  $1/2$

**Problem 18**

A  $3 \times 3 \times 3$  cube is made of 27 normal dice. Each die's opposite sides sum to 7. What is the smallest possible sum of all of the values visible on the 6 faces of the large cube?

一个  $3 \times 3 \times 3$  的立方体是由 27 个普通骰子组成的。每个骰子对面的两个数和为 7。

则大的立方体的 6 个面上可见的所有数之和的最小值是多少？

- (A) 60      (B) 72      (C) 84      (D) 90      (E) 96

**Problem 19**

Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside of the doghouse that Spot can reach?

Spot 的狗窝有一个边长为 1 码的正六边形底座。他被一根两码长的绳子拴在某个顶点上。问 Spot 可以到达的狗窝外区域的面积（以平方码为单位）是多少？

- (A)  $2\pi/3$       (B)  $2\pi$       (C)  $5\pi/2$       (D)  $8\pi/3$       (E)  $3\pi$

**Problem 20**

Points  $A, B, C, D, E$  and  $F$  lie, in that order, on  $\overline{AF}$ , dividing it into five segments, each of length 1. Point  $G$  is not on line  $AF$ . Point  $H$  lies on  $\overline{GD}$ , and point  $J$  lies on  $\overline{GF}$ . The line segments  $\overline{HC}$ ,  $\overline{JE}$ , and  $\overline{AG}$  are parallel. Find  $HC/JE$ .

点  $A, B, C, D, E$  和  $F$  以此顺序位于线段  $\overline{AF}$  上，将线段  $\overline{AF}$  分成 5 根长度均为 1 的线段。点  $G$  不在直线  $AF$  上。点  $H$  在  $\overline{GD}$  上，点  $J$  在  $\overline{GF}$  上。线段  $\overline{HC}$ ,  $\overline{JE}$ , 以及  $\overline{AG}$  相互平行。求  $HC/JE$ 。

- (A)  $5/4$       (B)  $4/3$       (C)  $3/2$       (D)  $5/3$       (E) 2

**Problem 21**

The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

某 8 个整数的平均数，中位数，唯一的众数和极差都等于 8。那么这 8 个整数中最大的整数可以是多少？

- (A) 11      (B) 12      (C) 13      (D) 14      (E) 15

**Problem 22**

A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

编号为 1 到 100 的一组瓷砖通过以下操作重复修改：移除所有编号为完全平方数的瓷砖，然后对剩余的瓷砖从 1 开始重新连续编号。问必须执行多少次这样的操作才能使得这组瓷砖最后只剩下一块？

- (A) 10      (B) 11      (C) 18      (D) 19      (E) 20

**Problem 23**

Points  $A, B, C$  and  $D$  lie on a line, in that order, with  $AB = CD$  and  $BC = 12$ . Point  $E$  is not on the line, and  $BE = CE = 10$ . The perimeter of  $\triangle AED$  is twice the perimeter of  $\triangle BEC$ . Find  $AB$ .

点  $A, B, C$  和  $D$  以此顺序位于一条直线上，满足  $AB = CD$  以及  $BC = 12$ 。点  $E$  不在这条直线上，且  $BE = CE = 10$ 。 $\triangle AED$  的周长是  $\triangle BEC$  周长的 2 倍。求  $AB$  的长度。

- (A)  $15/2$       (B) 8      (C)  $17/2$       (D) 9      (E)  $19/2$

**Problem 24**

Tina randomly selects two distinct numbers from the set  $\{1, 2, 3, 4, 5\}$ , and Sergio randomly selects a number from the set  $\{1, 2, \dots, 10\}$ . What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

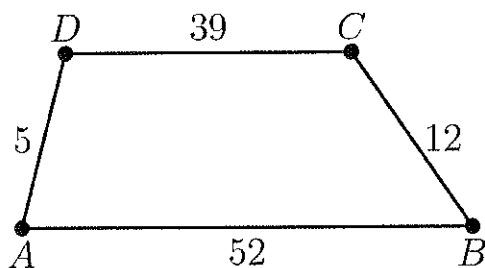
Tina 从集合  $\{1, 2, 3, 4, 5\}$  中随机选择 2 个不同的数，Sergio 从集合  $\{1, 2, \dots, 10\}$  中随机选择一个数。问 Sergio 的数大于 Tina 所选的两个数之和的概率是多少？

- (A)  $2/5$       (B)  $9/20$       (C)  $1/2$       (D)  $11/20$       (E)  $24/25$

**Problem 25**

In trapezoid  $ABCD$  with bases  $AB$  and  $CD$ , we have  $AB = 52$ ,  $BC = 12$ ,  $CD = 39$ , and  $DA = 5$  (diagram not to scale). The area of  $ABCD$  is

在底边为  $AB$  和  $CD$  的梯形  $ABCD$  中，我们有  $AB = 52$ ,  $BC = 12$ ,  $CD = 39$ , 以及  $DA = 5$  (图没有按照比例画)。那么  $ABCD$  的面积是



- (A) 182      (B) 195      (C) 210      (D) 234      (E) 260