

2002 AMC10B 试题

Problem 1

The ratio $\frac{2^{2001} \cdot 3^{2003}}{6^{2002}}$ is:

$\frac{2^{2001} \cdot 3^{2003}}{6^{2002}}$ 的比值是:

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{2}$

Problem 2

For the nonzero numbers a , b , and c , define $(a, b, c) = \frac{abc}{a + b + c}$. Find $(2, 4, 6)$.

a, b 和 c 均不为 0, 定义 $(a, b, c) = \frac{abc}{a + b + c}$. 求 $(2, 4, 6)$ 。

- (A) 1 (B) 2 (C) 4 (D) 6 (E) 24

Problem 3

The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \dots, 999999999\}$ is a 9-digit number M , all of whose digits are distinct. The number M does not contain the digit

集合 $\{9, 99, 999, 9999, \dots, 999999999\}$ 中的 9 个数的算术平均值是个 9 位数 M ,

且 M 的各个位上数字均不同。那么 M 不包含数字

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Problem 4

What is the value of $(3x - 2)(4x + 1) - (3x - 2)4x + 1$ when $x = 4$?

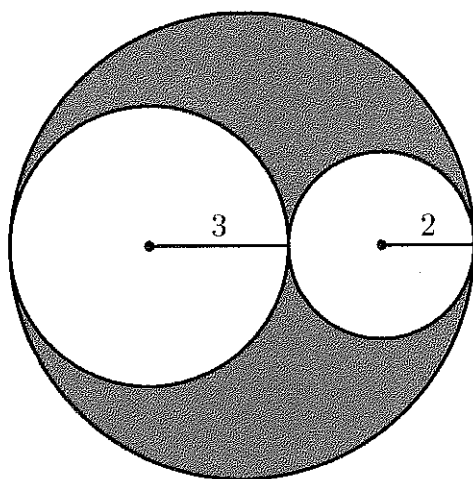
当 $x = 4$ 时, $(3x - 2)(4x + 1) - (3x - 2)4x + 1$ 的值是多少?

- (A) 0 (B) 1 (C) 10 (D) 11 (E) 12

Problem 5

Circles of radius 2 and 3 are externally tangent and are circumscribed by a third circle, as shown in the figure. Find the area of the shaded region.

半径为 2 和 3 的两个圆相外切，且和第三个圆相内切，如图所示。求阴影部分的面积。



- (A) 3π (B) 4π (C) 6π (D) 9π (E) 12π

Problem 6

For how many positive integers n is $n^2 - 3n + 2$ a prime number?

- (A) none (B) one (C) two (D) more than two, but finitely many (E) infinitely many

有多少个正整数 n ，使得 $n^2 - 3n + 2$ 是个质数？

- (A) 0 个 (B) 1 个 (C) 2 个 (D) 多于 2 个，但是有限多个 (E) 无

限多个

Problem 7

Let n be a positive integer such that $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ is an integer. Which of the following statements is **not** true?

- (A) 2 divides n (B) 3 divides n (C) 6 divides n (D) 7 divides n (E) $n > 84$

n 是个正整数，使得 $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ 是个整数。下面哪句话不正确？

- (A) 2 整除 n (B) 3 整除 n (C) 6 整除 n (D) 7 整除 n (E) $n > 84$

Problem 8

Suppose July of year N has five Mondays. Which of the following must occur five times in the August of year N ? (Note: Both months have 31 days.)

- (A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday

假设年份为 N 的这一年的七月有 5 个周一。那么下面哪个必定会在这一年的八月出现 5 次?(注意: 这两个月都有 31 天)

- (A) 周一 (B) 周二 (C) 周三 (D) 周四 (E) 周五

Problem 9

Using the letters A, M, O, S , and U , we can form five-letter "words". If these "words" are arranged in alphabetical order, then the "word" $USAMO$ occupies position

使用字母 A, M, O, S 和 U , 我们可以形成多个 5 个字母的“单词”。如果把这些“单词”按照字母顺序表的顺序排列, 那么“单词” $USAMO$ 占据第几个位置?

- (A) 112 (B) 113 (C) 114 (D) 115 (E) 116

Problem 10

Suppose that a and b are nonzero real numbers, and that the

equation $x^2 + ax + b = 0$ has solutions a and b . What is the pair (a, b) ?

假设 a 和 b 均为非零实数, 且方程 $x^2 + ax + b = 0$ 的解为 a 和 b 。那么实数对 (a, b) 是多少?

- (A) $(-2, 1)$ (B) $(-1, 2)$ (C) $(1, -2)$ (D) $(2, -1)$ (E) $(4, 4)$

Problem 11

The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

三个连续正整数之积是它们的和的 8 倍。那么它们的平方和是多少?

- (A) 50 (B) 77 (C) 110 (D) 149 (E) 194

Problem 12

For which of the following values of k does the equation $\frac{x-1}{x-2} = \frac{x-k}{x-6}$ have no solution for x ?

k 为下面哪个值时, 方程 $\frac{x-1}{x-2} = \frac{x-k}{x-6}$ 中的 x 无解?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 13

Find the value(s) of x such that $8xy - 12y + 2x - 3 = 0$ is true for all values of y .

求使得方程 $8xy - 12y + 2x - 3 = 0$ 对所有 y 值都成立的 x 的值。

- (A) $\frac{2}{3}$ (B) $\frac{3}{2}$ or $-\frac{1}{4}$ (C) $-\frac{2}{3}$ or $-\frac{1}{4}$ (D) $\frac{3}{2}$ (E) $-\frac{3}{2}$ or $-\frac{1}{4}$

Problem 14

The number $25^{64} \cdot 64^{25}$ is the square of a positive integer N . In decimal representation, the sum of the digits of N is

数字 $25^{64} \cdot 64^{25}$ 是一个正整数 N 的平方。那么 N 的十进制表示的所有位上数字之

和是

- (A) 7 (B) 14 (C) 21 (D) 28 (E) 35

Problem 15

The positive integers A , B , $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is

正整数 A , B , $A - B$, 和 $A + B$ 均为质数。那么这 4 个质数的和

- (A) 是个偶数 (B) 能被 3 整除 (C) 能被 5 整除 (D) 能被 7 整除 (E)

是个质数

Problem 16

For how many integers n is $\frac{n}{20-n}$ the square of an integer?

有多少个整数 n , 使得 $\frac{n}{20-n}$ 是某个整数的平方?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 10

Problem 17

A regular octagon $ABCDEFGH$ has sides of length two. Find the area of $\triangle ADG$.

一个正八边形 $ABCDEFGH$ 的边长为2, 求 $\triangle ADG$ 的面积。

- (A) $4 + 2\sqrt{2}$ (B) $6 + \sqrt{2}$ (C) $4 + 3\sqrt{2}$ (D) $3 + 4\sqrt{2}$ (E) $8 + \sqrt{2}$

Problem 18

Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

在平面内画了4个不同的圆。则最多有多少个这样的点, 满足至少有2个圆相交于此点?

- (A) 8 (B) 9 (C) 10 (D) 12 (E) 16

Problem 19

Suppose that $\{a_n\}$ is an arithmetic sequence

with $a_1 + a_2 + \cdots + a_{100} = 100$ and $a_{101} + a_{102} + \cdots + a_{200} = 200$. What is the value of $a_2 - a_1$?

假设数列 $\{a_n\}$ 是个等差数列, 满足 $a_1 + a_2 + \cdots + a_{100} = 100$ 且

$a_{101} + a_{102} + \cdots + a_{200} = 200$. 那么 $a_2 - a_1$ 的值是多少?

- (A) 0.0001 (B) 0.001 (C) 0.01 (D) 0.1 (E) 1

Problem 20

Let a , b , and c be real numbers such

that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

a, b , 和 c 均为实数, 满足 $a - 7b + 8c = 4$ 以及 $8a + 4b - c = 7$. 那么

$a^2 - b^2 + c^2$ 等于

- (A) 0 (B) 1 (C) 4 (D) 7 (E) 8

Problem 21

Andy's lawn has twice as much area as Beth's lawn and three times as much as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as

Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

Andy 的草坪面积是 Beth 的草坪面积的两倍, 是 Carlos 的三倍。Carlos 的割草机割草速度是 Beth 的一半, 是 Andy 的三分之一。如果她们同时开始割草, 谁会先割完?

- (A) Andy (B) Beth (C) Carlos (D) Andy 和 Carlos 同时先割完 (E) 三人平局

Problem 22

Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of the legs OX and OY , respectively. Given $XN = 19$ and $YM = 22$, find XY .

$\triangle XOY$ 是个直角三角形, 其中 $m\angle XOY = 90^\circ$ 。点 M 和 N 分别是直角边 OX 和 OY 的中点。已知 $XN = 19$, $YM = 22$, 求 XY 。

- (A) 24 (B) 26 (C) 28 (D) 30 (E) 32

Problem 23

Let $\{a_k\}$ be a sequence of integers such

that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$, for all positive integers m and n . Then a_{12} is

$\{a_k\}$ 是整数数列, 满足 $a_1 = 1$ 以及 $a_{m+n} = a_m + a_n + mn$, 对所有的正整数 m 和 n 都成立。那么 a_{12} 等于

- (A) 45 (B) 56 (C) 67 (D) 78 (E) 89

Problem 24

Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

摩天轮上的乘客绕着垂直面内的一个圆旋转。某种特定的轮子半径为 20 英尺, 并以每分钟一圈的恒定速度旋转。那么乘客从车轮底部旋转到离车轮底部垂直距离为 10 英尺的点需要多少秒?

- (A) 5 (B) 6 (C) 7.5 (D) 10 (E) 15

Problem 25

When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

若在一整数数列的尾部添加数字 15, 则整个数列的平均值增加 2; 若继续将数字 1 添加到拓展后的数列的尾部, 则拓展后数列的平均值再减少 1. 问原始数列里有多少个整数?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8