

7-04

2003 AMC10A 试题

Problem 1

What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

前 2003 个偶正整数之和与前 2003 个奇正整数之和的差是多少?

- (A) 0 (B) 1 (C) 2 (D) 2003 (E) 4006

Problem 2

Members of the Rockham Soccer League buy socks and T-shirts. Socks cost \$4 per pair and each T-shirt costs \$5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is \$2366, how many members are in the League?

Rockham 足球联盟的成员购买袜子和 T 恤。袜子每双 4 美元，每件 T 恤比一双袜子贵 5 美元。每个成员在主场比赛需要一双袜子和一件衬衫，客场比赛需要另

一双袜子和一件衬衫。如果总费用是 2366 美元，那么联盟中有多少成员?

- (A) 77 (B) 91 (C) 143 (D) 182 (E) 286

Problem 3

A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of this box. What percent of the original volume is removed?

一个实心盒子的尺寸是 15 厘米乘 10 厘米乘 8 厘米。现在在这个盒子的每个角落移除一个边长为 3 厘米的正方体后，形成一个新的立体图形。则原来体积的百分之多少被移除了?

- (A) 4.5 (B) 9 (C) 12 (D) 18 (E) 24

Problem 4

It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to her home along the same route. What is her average speed, in km/hr, for the round trip?

Mary 从家步行上山 1 千米到学校需要 30 分钟,但从学校沿着同一条路线步行到家只需要 10 分钟。那么她往返的平均速度是多少千米每小时?

- (A) 3 (B) 3.125 (C) 3.5 (D) 4 (E) 4.5

Problem 5

Let d and e denote the solutions of $2x^2 + 3x - 5 = 0$. What is the value of $(d - 1)(e - 1)$?

d 和 e 是方程 $2x^2 + 3x - 5 = 0$ 的两个根, 则 $(d - 1)(e - 1)$ 是多少?

- (A) $-\frac{5}{2}$ (B) 0 (C) 3 (D) 5 (E) 6

Problem 6

Define $x \heartsuit y$ to be $|x - y|$ for all real numbers x and y . Which of the following statements is not true?

定义 $x \heartsuit y$ 为 $|x - y|$, x 和 y 为所有实数。下面哪句话不正确?

- (A) $x \heartsuit y = y \heartsuit x$, 对所有的 x 和 y
(B) $2(x \heartsuit y) = (2x) \heartsuit (2y)$, 对所有的 x 和 y
(C) $x \heartsuit 0 = x$, 对所有的 x
(D) $x \heartsuit x = 0$, 对所有的 x
(E) $x \heartsuit y > 0$, 若 $x \neq y$

Problem 7

How many non-congruent triangles with perimeter 7 have integer side lengths?

有多少个周长为 7 非全等的三角形的三条边长均为整数?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 8

What is the probability that a randomly drawn positive factor of 60 is less than 7?

从 60 的所有正因子中随机挑选一个，问这个因子小于 7 的概率是多少？

- (A) $\frac{1}{10}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

Problem 9

Simplify

$$\sqrt[3]{x \sqrt[3]{x \sqrt[3]{x \sqrt{x}}}}$$

化简

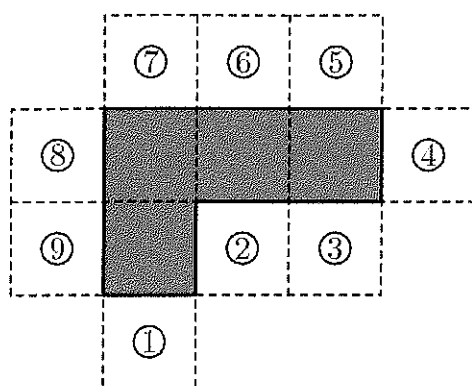
$$\sqrt[3]{x \sqrt[3]{x \sqrt[3]{x \sqrt{x}}}}$$

- (A) $\sqrt{x}$ (B) $\sqrt[3]{x^2}$ (C) $\sqrt[27]{x^2}$ (D) $\sqrt[54]{x}$ (E) $\sqrt[81]{x^{80}}$

Problem 10

The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?

图中实线包围的多边形由 4 个边到边连接的全等正方形组成。在如图所示的九个位置中选择一个位置，再接入另一个全等的正方形。问形成的 9 种不同的多边形中，有多少个可以折叠成一个缺少一个面的正方体？



- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 11

The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A + M + C$?

两个 5 位数 $AMC10$ 和 $AMC12$ 之和是 123422。那么 $A + M + C$ 是多少?

- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

Problem 12

A point (x, y) is randomly picked from inside the rectangle with

vertices $(0, 0)$, $(4, 0)$, $(4, 1)$, and $(0, 1)$. What is the probability that $x < y$?

从顶点为 $(0, 0)$, $(4, 0)$, $(4, 1)$ 和 $(0, 1)$ 的矩形中随机选择一个点 (x, y) 。则 $x < y$ 的概率是多少?

- (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) $\frac{3}{4}$

Problem 13

The sum of three numbers is 20. The first is four times the sum of the other two. The second is seven times the third. What is the product of all three?

三个数之和为 20。第一个数是其他两个数之和的 4 倍。第二个数是第三个数的 7 倍。那么这三个数的乘积是多少?

- (A) 28 (B) 40 (C) 100 (D) 400 (E) 800

Problem 14

Let n be the largest integer that is the product of exactly 3 distinct prime numbers d , e , and $10d + e$, where d and e are single digits. What is the sum of the digits of n ?

n 是 3 个不同的质数 d , e , 和 $10d + e$ 的乘积所能得到的最大的整数, 其中 d 和 e 都

是一位数。那么 n 的所有位上数字之和是多少?

- (A) 12 (B) 15 (C) 18 (D) 21 (E) 24

Problem 15

What is the probability that an integer in the set $\{1, 2, 3, \dots, 100\}$ is divisible by 2 and not divisible by 3?

集合 $\{1, 2, 3, \dots, 100\}$ 内的一个整数能被 2 整除但不能被 3 整除的概率是多少?

- (A) $\frac{1}{6}$ (B) $\frac{33}{100}$ (C) $\frac{17}{50}$ (D) $\frac{1}{2}$ (E) $\frac{18}{25}$

Problem 16

What is the units digit of 13^{2003} ?

13^{2003} 的个位数字是多少

- (A) 1 (B) 3 (C) 7 (D) 8 (E) 9

Problem 17

The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?

一个等边三角形的周长 (单位:英寸) 数值上等于它的外接圆的面积 (单位:平方英寸)。则这个圆的半径是多少英寸?

- (A) $\frac{3\sqrt{2}}{\pi}$ (B) $\frac{3\sqrt{3}}{\pi}$ (C) $\sqrt{3}$ (D) $\frac{6}{\pi}$ (E) $\sqrt{3}\pi$

Problem 18

What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0 \quad ?$$

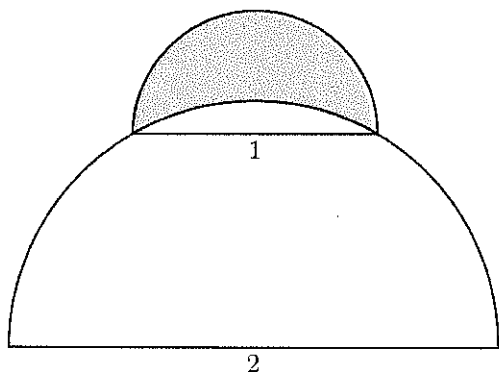
方程 $\frac{2003}{2004}x + 1 + \frac{1}{x} = 0$ 的所有根的倒数之和是多少?

- (A) $-\frac{2004}{2003}$ (B) -1 (C) $\frac{2003}{2004}$ (D) 1 (E) $\frac{2004}{2003}$

Problem 19

A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.

如图所示, 一个直径为 1 的半圆位于另一个直径为 2 的半圆的顶部。在小半圆内部而在大半圆外部的阴影部分图形被称作新月形。求这个新月形的面积。



- (A) $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{4} - \frac{1}{12}\pi$ (C) $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ (D) $\frac{\sqrt{3}}{4} + \frac{1}{24}\pi$ (E) $\frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

Problem 20

A base-10 three digit number n is selected at random. Which of the following is closest to the probability that the base-9 representation and the base-11 representation of n are both three-digit numerals?

随机选择一个十进制的三位数 n 。那么 n 的 9 进制和 11 进制也都是三位数的概率最接近下面哪一个?

- (A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6 (E) 0.7

Problem 21

Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

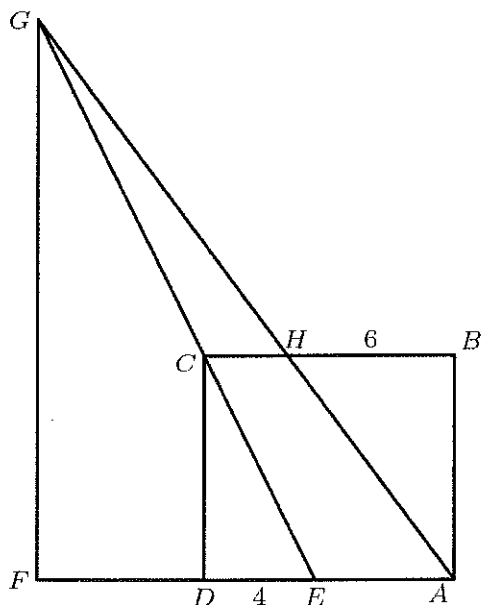
Pat 将从一个只装有巧克力片味、燕麦味和花生酱味这三种口味饼干的托盘中选择六块饼干。已知托盘上这三种饼干每种都至少有六块。问有多少种选择这六块饼干的方法?

- (A) 22 (B) 25 (C) 27 (D) 28 (E) 729

Problem 22

In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G , and F is on line AD with $GF \perp AF$. Find the length of GF .

在矩形 $ABCD$ 中, 我们有 $AB = 8$, $BC = 9$, 点 H 在 BC 上, 且 $BH = 6$ 。点 E 在 AD 上且 $DE = 4$ 。直线 EC 交直线 AH 于 G , 点 F 在直线 AD 上, 且 $GF \perp AF$ 。求 GF 的长度。

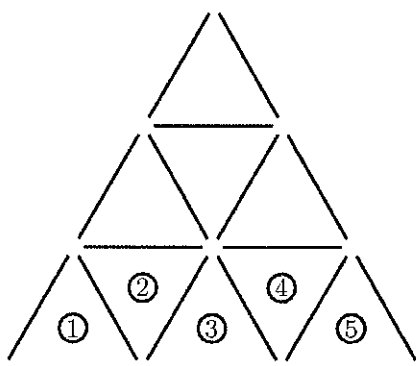


- (A) 16 (B) 20 (C) 24 (D) 28 (E) 30

Problem 23

A large equilateral triangle is constructed by using toothpicks to create rows of small equilateral triangles. For example, in the figure we have 3 rows of small congruent equilateral triangles, with 5 small triangles in the base row. How many toothpicks would be needed to construct a large equilateral triangle if the base row of the triangle consists of 2003 small equilateral triangles?

通过用牙签创建若干行小的等边三角形，最终构成一个大的等边三角形。例如，在下图中，我们有 3 行小的全等等边三角形，其中底行有 5 个小三角形。如果底行现在有 2003 个小的等边三角形，那么需要多少根牙签才能构成一个大的等边三角形？



- (A) 1,004,004 (B) 1,005,006 (C) 1,507,509 (D) 3,015,018 (E) 6,021,018

Problem 24

Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

Sally 有五张红牌，牌面上的数字分别是 1 到 5，以及四张蓝牌，牌面上的数字分别是 3 到 6。她将卡片堆叠起来，使颜色交替，并使每张红牌上的数字能够整除每张与它相邻蓝牌上的数字。问中间三张牌上的数字之和是多少？

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

Problem 25

Let n be a 5-digit number, and let q and r be the quotient and the remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?

n 是个5位数, q 和 r 分别是 n 除以100所得商和余数。则 n 有多少个值, 可以使得 $q + r$ 能被11整除?

- (A) 8180 (B) 8181 (C) 8182 (D) 9000 (E) 9090