

2003 AMC10B 试题

Problem 1

Which of the following is the same as

$$\frac{2 - 4 + 6 - 8 + 10 - 12 + 14}{3 - 6 + 9 - 12 + 15 - 18 + 21}?$$

以下选项哪个等于

$$\frac{2 - 4 + 6 - 8 + 10 - 12 + 14}{3 - 6 + 9 - 12 + 15 - 18 + 21}?$$

- (A) -1 (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) $\frac{14}{3}$

Problem 2

Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs \$1 more than a pink pill, and Al's pills cost a total of \$546 for the two weeks. How much does one green pill cost?

Al 患有一种病，叫做 algebritis，必须每天服用一粒绿色药丸和一粒粉色药丸，且服用 2 周。一粒绿色药丸比一粒粉色药丸贵 1 美元，Al 这两周的药丸总价格为 546 美元。问一粒绿色药丸多少钱？

- (A) \$7 (B) \$14 (C) \$19 (D) \$20 (E) \$39

Problem 3

The sum of 5 consecutive even integers is 4 less than the sum of the first 8 consecutive odd counting numbers. What is the smallest of the even integers?

5 个连续偶整数之和比前 8 个连续的奇正整数之和小 4。那么这些偶整数中最小的数是多少？

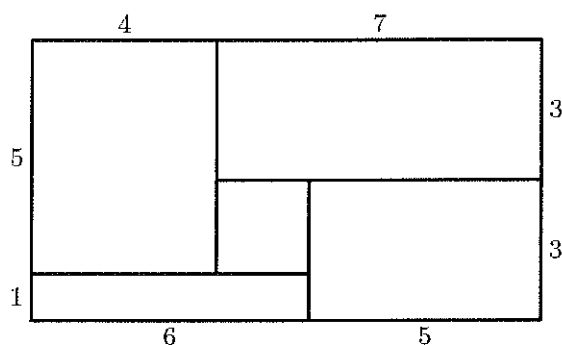
- (A) 6 (B) 8 (C) 10 (D) 12 (E) 14

Problem 4

Rose fills each of the rectangular regions of her rectangular flower bed with a different type of flower. The lengths, in feet, of the rectangular regions in her flower bed are as shown in the figure. She plants one flower per square foot in each region.

Asters cost \$1 each, begonias \$1.50 each, cannas \$2 each, dahlias \$2.50 each, and Easter lilies \$3 each. What is the least possible cost, in dollars, for her garden?

Rose 用不同类型的花填满了她矩形花坛的每个矩形区域。她的花坛中矩形区域的长度(英尺)如图所示。她在每个区域每平方英尺种一株花。紫苑每株 1 美元,秋海棠每株 1.5 美元,美人蕉每株 2 美元,大丽花每株 2.5 美元,复活节百合每株 3 美元。那么她的花园的最低成本是多少美元?



- (A) 108 (B) 115 (C) 132 (D) 144 (E) 156

Problem 5

Moe uses a mower to cut his rectangular 90-foot by 150-foot lawn. The swath he cuts is 28 inches wide, but he overlaps each cut by 4 inches to make sure that no grass is missed. He walks at the rate of 5000 feet per hour while pushing the mower. Which of the following is closest to the number of hours it will take Moe to mow the lawn?

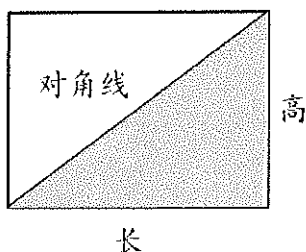
Moe 使用割草机修剪他的90英尺乘150英尺的矩形草坪。他修剪的草带宽度为28英寸,但他所修剪的每一块草皮会重叠4英寸,以确保不会遗漏任何草。他推着割草机行走的速度是每小时 5000 英尺。以下哪项最接近 Moe 修剪草坪所需的小时数?

- (A) 0.75 (B) 0.8 (C) 1.35 (D) 1.5 (E) 3

Problem 6

Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is 4 : 3. The horizontal length of a "27-inch" television screen is closest, in inches, to which of the following?

许多电视屏幕都是矩形，且它们的尺寸是用对角线长度来度量的。标准电视屏幕的水平长度与高度之比为4:3。一个“27-英寸”的电视屏幕的水平长度最接近以下哪个数字（以英寸为单位）？



- (A) 20 (B) 20.5 (C) 21 (D) 21.5 (E) 22

Problem 7

The symbolism $\lfloor x \rfloor$ denotes the largest integer not exceeding x . For example, $\lfloor 3 \rfloor = 3$, and $\lfloor 9/2 \rfloor = 4$.

Compute $\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{16} \rfloor$.

符号 $\lfloor x \rfloor$ 表示取不超过 x 的最大整数。例如， $\lfloor 3 \rfloor = 3$, $\lfloor 9/2 \rfloor = 4$ 。计算

$\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{16} \rfloor$.

- (A) 35 (B) 38 (C) 40 (D) 42 (E) 136

Problem 8

The second and fourth terms of a geometric sequence are 2 and 6. Which of the following is a possible first term?

一个等比数列的第2项和第4项分别是2和6。则第一项可能是下面哪个？

- (A) $-\sqrt{3}$ (B) $-\frac{2\sqrt{3}}{3}$ (C) $-\frac{\sqrt{3}}{3}$ (D) $\sqrt{3}$ (E) 3

Problem 9

Find the value of x that satisfies the equation $25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$.

求满足方程 $25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$ 的 x 的值。

- (A) 2 (B) 3 (C) 5 (D) 6 (E) 9

Problem 10

Nebraska, the home of the AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by four digits. Each new license plate consists of the three letters followed by three digits. By how many times is the number of possible license plates increased?

AMC 总部所在地内布拉斯加州改变了车牌方案。每个旧车牌由一个字母和四个数字组成。每个新车牌由三个字母和三个数字组成。那么可能的车牌数量是原来的多少倍?

- (A) $\frac{26}{10}$ (B) $\frac{26^2}{10^2}$ (C) $\frac{26^2}{10}$ (D) $\frac{26^3}{10^3}$ (E) $\frac{26^3}{10^2}$

Problem 11

A line with slope 3 intersects a line with slope 5 at point $(10, 15)$. What is the distance between the x -intercepts of these two lines?

一条斜率为 3 的直线和一条斜率为 5 的直线交于点 $(10, 15)$ 。那么这 2 条直线的 x 截距之间的距离是多少?

- (A) 2 (B) 5 (C) 7 (D) 12 (E) 20

Problem 12

Al, Betty, and Clare split \$1000 among them to be invested in different ways. Each begins with a different amount. At the end of one year, they have a total of \$1500. Betty and Clare have both doubled their money, whereas Al has managed to lose \$100. What was Al's original portion?

Al、Betty 和 Clare 三个人分 1000 美元，并各自以不同的方式进行投资。每个人分得的初始金额不同。一年后，他们总共有 1500 美元。Betty 和 Clare 的钱都翻倍了，而 Al 损失了 100 美元。问 Al 的初始金额是多少美元?

- (A) \$250 (B) \$350 (C) \$400 (D) \$450 (E) \$500

Problem 13

Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x . For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1 + 2 + 3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

令 $\clubsuit(x)$ 表示正整数 x 所有位上数字之和。例如, $\clubsuit(8) = 8$,
 $\clubsuit(123) = 1 + 2 + 3 = 6$ 。那么有多少个两位数 x , 满足 $\clubsuit(\clubsuit(x)) = 3$?

(A) 3 (B) 4 (C) 6 (D) 9 (E) 10

Problem 14

Given that $3^8 \cdot 5^2 = a^b$, where both a and b are positive integers, find the smallest possible value for $a + b$.

已知 $3^8 \cdot 5^2 = a^b$, 其中 a 和 b 均为正整数, 求 $a + b$ 的最小可能值。

- (A) 25 (B) 34 (C) 351 (D) 407 (E) 900

Problem 15

There are 100 players in a single tennis tournament. The tournament is single elimination, meaning that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The tournament continues until only one player remains unbeaten. The total number of matches played is

一场网球锦标赛有 100 名选手。锦标赛是单淘汰赛, 意味着输掉比赛的球员将被淘汰。在第一轮比赛中, 实力最强的 28 名球员给予首轮轮空的优待 (第一轮不用参加), 剩下的 72 名球员将配对进行比赛。每轮比赛结束后, 剩下的球员将参加下一轮比赛。比赛一直持续到只有一名选手保持不败。那么比赛的总场数

(A) 是一个质数 (B) 能被 2 整除 (C) 能被 5 整除 (D) 能被 7 整除 (E) 能被 11 整除

Problem 16

A restaurant offers three desserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a dessert. What is the least number of main courses that a restaurant should offer so that a customer could have a different dinner each night in the year 2003?

一家餐厅提供三种甜点，并且开胃菜的数量是主菜的两倍。一顿晚餐由一道开胃菜、一道主菜和一个甜点组成。为了让顾客在 2003 年这一年中每天晚上都能吃到不同的晚餐，餐厅应该至少提供多少道主菜？

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Problem 17

An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius? (Note: a cone with radius r and height h has volume $\pi r^2 h/3$ and a sphere with radius r has volume $4\pi r^3/3$.)

某个冰淇淋蛋筒由一个香草冰淇淋球体和一个直径与球体直径相同的正圆锥体形状的蛋筒组成。如果冰淇淋融化，它将恰好能填满蛋筒。假设融化了的冰淇淋的体积是冷冻冰淇淋总体积的 75%。问圆锥体蛋筒的高度与它的半径之比是多少？（注意：半径为 r ，高度为 h 的圆锥体的体积为 $\pi r^2 h/3$ ，半径为 r 的球体体积为 $4\pi r^3/3$ 。）

(A) 2 : 1 (B) 3 : 1 (C) 4 : 1 (D) 16 : 3 (E) 6 : 1

Problem 18

What is the largest integer that is a divisor

of $(n+1)(n+3)(n+5)(n+7)(n+9)$ for all positive even integers n ?

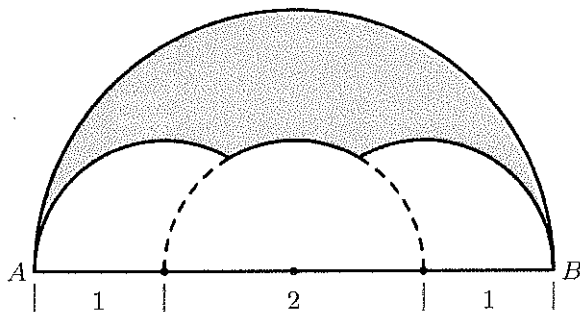
n 是所有的正偶数, 则 $(n+1)(n+3)(n+5)(n+7)(n+9)$ 的最大因子是
少?

- (A) 3 (B) 5 (C) 11 (D) 15 (E) 165

Problem 19

Three semicircles of radius 1 are constructed on diameter $\overline{AB}$ of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?

在半径为 2 的半圆的直径 $\overline{AB}$ 上作三个半径为 1 的半圆。三个小的半圆的圆心将 $\overline{AB}$ 分成四等份, 如图所示。那么在大半圆内部而在三个小半圆外部的阴影部分区域的面积是多少?



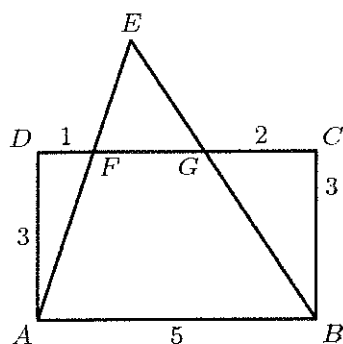
- (A) $\pi - \sqrt{3}$ (B) $\pi - \sqrt{2}$ (C) $\frac{\pi + \sqrt{2}}{2}$ (D) $\frac{\pi + \sqrt{3}}{2}$ (E) $\frac{7}{6}\pi - \frac{\sqrt{3}}{2}$

Problem 20

In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E . Find the area of $\triangle AEB$.

在矩形 $ABCD$ 中, $AB=5, BC=3$. 点 F 和 G 位于 $\overline{CD}$ 上, 满足 $DF = 1, GC = 2$ 。

直线 AF 交 BG 于 E 。求 $\triangle AEB$ 的面积。



- (A) 10 (B) $\frac{21}{2}$ (C) 12 (D) $\frac{25}{2}$ (E) 15

Problem 21

A bag contains two red beads and two green beads. You reach into the bag and pull out a bead, replacing it with a red bead regardless of the color you pulled out. What is the probability that all beads in the bag are red after three such replacements?

一个袋子里有两个红色的珠子和两个绿色的珠子。你把手伸进袋子，拿出一颗珠子。不管你拿出的是什么颜色，将这颗珠子换成一颗红色珠子。问经过三次更换后，袋子里的所有珠子都是红色的概率是多少？

- (A) $\frac{1}{8}$ (B) $\frac{5}{32}$ (C) $\frac{9}{32}$ (D) $\frac{3}{8}$ (E) $\frac{7}{16}$

Problem 22

A clock chimes once at 30 minutes past each hour and chimes on the hour according to the hour. For example, at 1PM there is one chime and at noon and midnight there are twelve chimes. Starting at 11:15AM on February 26, 2003, on what date will the 2003rd chime occur?

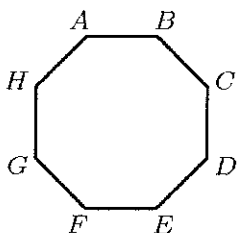
某个时钟在每个整点过 30 分钟时会报时一次，在每个整点报时的次数等于这个整点的数字。例如，下午 1 点时会报时一次，且在中午 12 点和凌晨 12 点时，均报时 12 次。则从 2003 年 2 月 26 日上午 11:15 开始，第 2003 次报时出现在哪个日期？

- (A) 3 月 8 日 (B) 3 月 9 日 (C) 3 月 10 日 (D) 3 月 20 日 (E) 3 月 21 日

Problem 23

A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of the rectangle $ABEF$?

一个正八边形 $ABCDEFGH$ 的面积为 1 平方单位。那么长方形 $ABEF$ 的面积是多少?



- (A) $1 - \frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{4}$ (C) $\sqrt{2} - 1$ (D) $\frac{1}{2}$ (E) $\frac{1 + \sqrt{2}}{4}$

Problem 24

The first four terms in an arithmetic sequence are $x + y$, $x - y$, xy , and x/y , in that order. What is the fifth term?

一个等差数列的前四项依次为 $x + y$, $x - y$, xy , 和 x/y , 那么第五项是多少?

- (A) $-\frac{15}{8}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{27}{20}$ (E) $\frac{123}{40}$

Problem 25

How many distinct four-digit numbers are divisible by 3 and have 23 as their last two digits?

有多少个不同的四位数, 能被 3 整除且最后两位上的数字是 23?

- (A) 27 (B) 30 (C) 33 (D) 81 (E) 90