

2004 AMC10A 试题

Problem 1

You and five friends need to raise 1500 dollars in donations for a charity, dividing the fundraising equally. How many dollars will each of you need to raise?

你和五个朋友需要为慈善机构筹集 1500 美元的捐款，并将筹款任务平均分配。

那么你们每个人需要筹集多少美元？

- (A) 250 (B) 300 (C) 1500 (D) 7500 (E) 9000

Problem 2

For any three real numbers a , b , and c , with $b \neq c$, the operation $\otimes$ is defined

by: $\otimes(a, b, c) = \frac{a}{b-c}$. What is $\otimes(\otimes(1, 2, 3), \otimes(2, 3, 1), \otimes(3, 1, 2))$?

对于任意三个实数 a, b, c , 且 $b \neq c$, 操作符 $\otimes$ 定义为 $\otimes(a, b, c) = \frac{a}{b-c}$. 那么 $\otimes(\otimes(1, 2, 3), \otimes(2, 3, 1), \otimes(3, 1, 2))$ 等于多少？

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) 0 (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

Problem 3

Alicia earns 20 dollars per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia's wages are used to pay local taxes?

Alicia 每小时挣 20 美元，其中扣除 1.45% 用来支付地方税。那么 Alicia 每小时工资中有多少美分用于支付地方税？

- (A) 0.0029 (B) 0.029 (C) 0.29 (D) 2.9 (E) 29

Problem 4

What is the value of x if $|x-1| = |x-2|$?

如果 $|x-1| = |x-2|$, 那么 x 的值是多少？

- (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) 2

Problem 5

A set of three points is randomly chosen from the grid shown. Each three point set has the same probability of being chosen. What is the probability that the points lie on the same straight line?

从所示网格中随机选择三个点组成一个集合。选择每个这样的三点集合的概率都相等。问这三个点位于同一条直线上的概率是多少？

• • •
• • •
• • •

- (A) $\frac{1}{21}$ (B) $\frac{1}{14}$ (C) $\frac{2}{21}$ (D) $\frac{1}{7}$ (E) $\frac{2}{7}$

Problem 6

Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and grand-daughters have no daughters?

Bertha 有 6 个女儿,没有儿子。她的一些女儿有 6 个女儿,其余的没有女儿。Bertha 一共有 30 个女儿和孙女,没有曾孙女。那么 Bertha 的女儿和孙女中有多少人没有女儿?

- (A) 22 (B) 23 (C) 24 (D) 25 (E) 26

Problem 7

A grocer stacks oranges in a pyramid-like stack whose rectangular base is 5 oranges by 8 oranges. Each orange above the first level rests in a pocket formed by four oranges below. The stack is completed by a single row of oranges. How many oranges are in the stack?

杂货商把桔子堆成金字塔形,长方形的底部是 5 个桔子乘 8 个桔子。第一层之上每层的每个桔子都放在下一层四个桔子组成的“口袋”里。这堆桔子最顶层是由一排桔子组成。那么这堆桔子一共有多少个桔子?

- (A) 96 (B) 98 (C) 100 (D) 101 (E) 134

Problem 8

A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token in the discard pile. The game ends when some player runs out of tokens. Players A , B , and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?

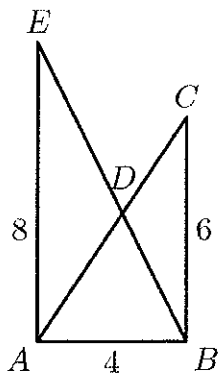
某个游戏按照以下规则使用令牌。在每一轮中，拥有最多令牌的玩家给其他玩家每人一个令牌，并同时将一个令牌放入弃牌堆中。当某个玩家用完全部令牌时，游戏结束。玩家 A , B , 和 C 一开始分别有 15、14 和 13 个令牌。那么这场比赛将有多少个回合？

- (A) 36 (B) 37 (C) 38 (D) 39 (E) 40

Problem 9

In the figure, $\angle EAB$ and $\angle ABC$ are right angles. $AB = 4$, $BC = 6$, $AE = 8$, and AC and BE intersect at D . What is the difference between the areas of $\triangle ADE$ and $\triangle BDC$?

如图， $\angle EAB$ 和 $\angle ABC$ 都是直角。 $AB = 4$, $BC = 6$, $AE = 8$, 且 AC 和 BE 交于 D 。那么 $\triangle ADE$ 和 $\triangle BDC$ 的面积之差是多少？



- (A) 2 (B) 4 (C) 5 (D) 8 (E) 9

Problem 10

Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

硬币 A 被掷了 3 次，硬币 B 被掷了 4 次。问掷这两枚标准硬币得到的正面次数相同的概率是多少？

- (A) $\frac{29}{128}$ (B) $\frac{23}{128}$ (C) $\frac{1}{4}$ (D) $\frac{35}{128}$ (E) $\frac{1}{2}$

Problem 11

A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

一家公司出售圆柱形罐子装的花生酱。市场研究表明，使用更宽的罐子将增加销售额。如果罐子的直径增加了 25%，而体积没有改变，那么高度必须减少多少百分比？

- (A) 10 (B) 25 (C) 36 (D) 50 (E) 60

Problem 12

Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

亨利汉堡天堂提供的汉堡有以下口味：番茄酱、芥末、蛋黄酱、西红柿、生菜、泡菜、奶酪和洋葱。顾客可以选择一块、两块或三块肉饼，以及任何口味或它们的组合（也可不选口味）。则可以订购多少种不同的汉堡包？

- (A) 24 (B) 256 (C) 768 (D) 40,320 (E) 120,960

Problem 13

At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?

在一次聚会上，每个男士和三个女士跳舞，且每个女士和两个男士跳舞。十二个男士参加了聚会。问有多少女士参加了聚会？

- (A) 8 (B) 12 (C) 16 (D) 18 (E) 24

Problem 14

The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average would be 21 cents. How many dimes does she have in her purse?

Paula 钱包中所有的便士、镍币、一角硬币和 25 分硬币的平均价值是 20 美分。

如果她的 25 分硬币再多一个，那么平均价值将是 21 美分。则她的钱包里有多少枚一角硬币？

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Problem 15

Given that $-4 \leq x \leq -2$ and $2 \leq y \leq 4$, what is the largest possible value of $\frac{x+y}{x}$?

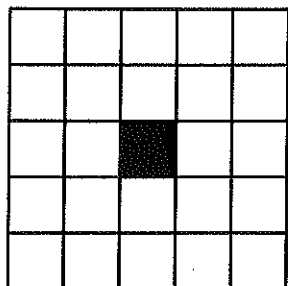
已知 $-4 \leq x \leq -2$ 且 $2 \leq y \leq 4$ ，那么表达式 $\frac{x+y}{x}$ 的最大可能值是多少？

- (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

Problem 16

The 5×5 grid shown contains a collection of squares with sizes from 1×1 to 5×5 . How many of these squares contain the black center square?

如下 5×5 的网格包含了一系列正方形，尺寸从 1×1 到 5×5 都有。这些正方形中，有多少个正方形包含了中间黑色正方形？



- (A) 12 (B) 15 (C) 17 (D) 19 (E) 20

Problem 17

Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

Brenda 和 Sally 在圆形跑道上从一条直径的两个相对的端点开始，朝相反的方向跑。她们第一次相遇时 Brenda 已经跑了 100 米。当 Sally 跑过第一次相遇地点 150 米后，他们再次相遇。每个女孩都以恒定的速度奔跑。这条赛道的长度是多少米？

- (A) 250 (B) 300 (C) 350 (D) 400 (E) 500

Problem 18

A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

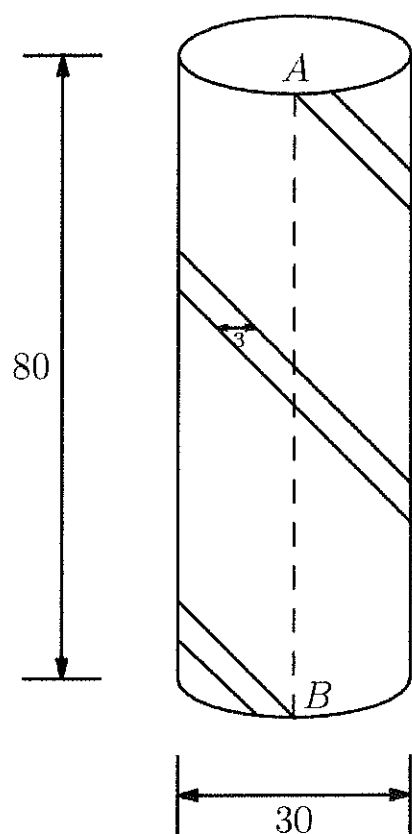
三个实数形成一个首项为 9 的等差数列。如果第二项加 2，第三项加 20，则结果的三个数字成等比数列。那么这个等比数列第三项的最小可能值是多少？

- (A) 1 (B) 4 (C) 36 (D) 49 (E) 81

Problem 19

A white cylindrical silo has a diameter of 30 feet and a height of 80 feet. A red stripe with a horizontal width of 3 feet is painted on the silo, as shown, making two complete revolutions around it. What is the area of the stripe in square feet?

一个白色圆柱形筒仓的直径为 30 英尺，高度为 80 英尺。如图所示，筒仓上绘制了一条水平宽度为 3 英尺的红色条纹，围绕筒仓旋转了两圈。问条纹的面积是多少平方英尺？

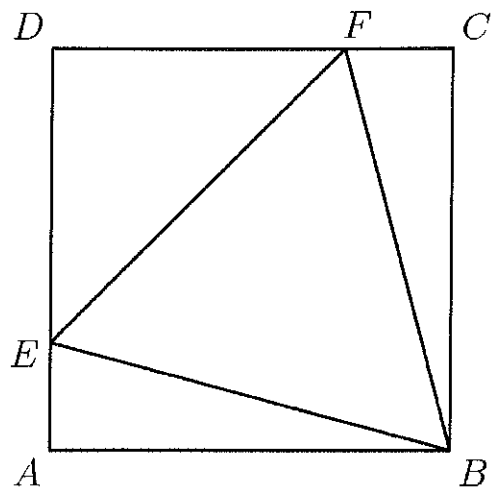


- (A) 120 (B) 180 (C) 240 (D) 360 (E) 480

Problem 20

Points E and F are located on square $ABCD$ so that $\triangle BEF$ is equilateral. What is the ratio of the area of $\triangle DEF$ to that of $\triangle ABE$?

点 E 和 F 在正方形 $ABCD$ 上，满足 $\triangle BEF$ 是个等边三角形。则 $\triangle DEF$ 的面积和 $\triangle ABE$ 的面积比值是多少？



- (A) $\frac{4}{3}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$ (D) 2 (E) $1 + \sqrt{3}$

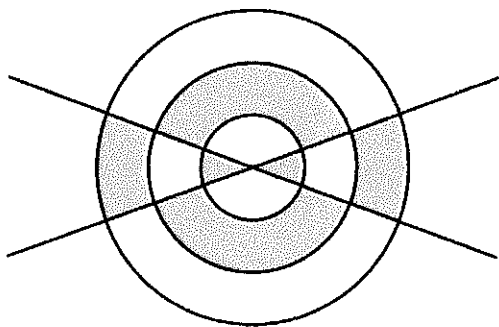
Problem 21

Two distinct lines pass through the center of three concentric circles of radii 3, 2, and

1. The area of the shaded region in the diagram is $\frac{8}{13}$ of the area of the unshaded region. What is the radian measure of the acute angle formed by the two lines?
(Note: π radians is 180 degrees.)

两条不同的线穿过半径为 3、2 和 1 的三个同心圆的中心。图中阴影区域的面积

是非阴影区域的面积的 $\frac{8}{13}$ 。这两条线形成的锐角是多少弧度？（注意： π 弧度是 180 度。）

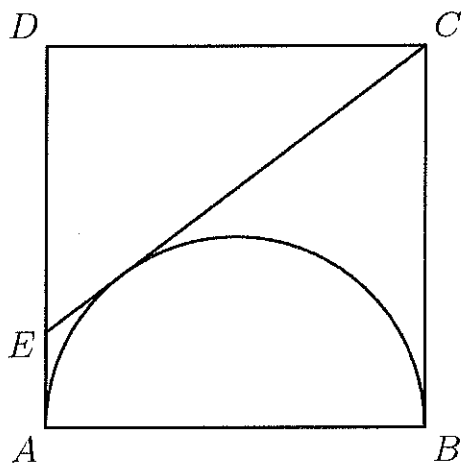


- (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{7}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{5}$ (E) $\frac{\pi}{4}$

Problem 22

Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E . What is the length of $\overline{CE}$?

正方形 $ABCD$ 的边长为 2. 直径为 $\overline{AB}$ 的半圆位于正方形内部, 从 C 引出的圆的切线交边 $\overline{AD}$ 于 E . 那么 $\overline{CE}$ 的长度是多少?

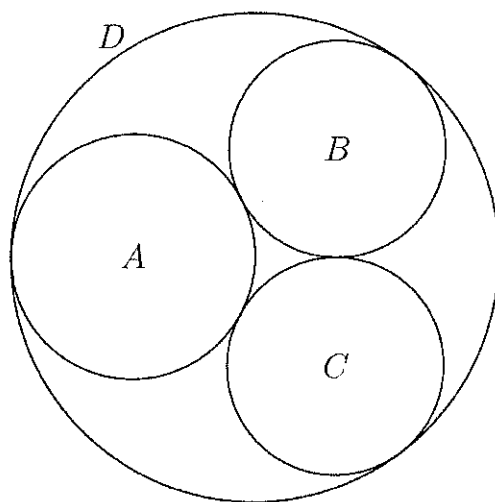


- (A) $\frac{2 + \sqrt{5}}{2}$ (B) $\sqrt{5}$ (C) $\sqrt{6}$ (D) $\frac{5}{2}$ (E) $5 - \sqrt{5}$

Problem 23

Circles A , B and C are externally tangent to each other, and internally tangent to circle D . Circles B and C are congruent. Circle A has radius 1 and passes through the center of D . What is the radius of circle B ?

圆 A , B 和 C 相互外切, 且均和圆 D 内切。圆 B 和圆 C 全等。圆 A 半径为 1, 且通过圆 D 的圆心。那么圆 B 的半径是多少?



- (A) $\frac{2}{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{7}{8}$ (D) $\frac{8}{9}$ (E) $\frac{1+\sqrt{3}}{3}$

Problem 24

Let f be a function with the following properties:

- (i) $f(1) = 1$, and
(ii) $f(2n) = n \cdot f(n)$ for any positive integer n .

What is the value of $f(2^{100})$?

f 是一个具有如下性质的函数:

- (i) $f(1) = 1$,
(ii) $f(2n) = n \cdot f(n)$, 对于任何正整数 n 都成立。

那么 $f(2^{100})$ 的值是多少?

- (A) 1 (B) 2^{99} (C) 2^{100} (D) 2^{4950} (E) 2^{9999}

Problem 25

Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

三个半径为 1 的球位于水平面上且相互外切。一个半径为 2 的球在它们之上。那

么从平面到大球顶部的距离是多少？

- (A) $3 + \frac{\sqrt{30}}{2}$ (B) $3 + \frac{\sqrt{69}}{3}$ (C) $3 + \frac{\sqrt{123}}{4}$ (D) $\frac{52}{9}$ (E) $3 + 2\sqrt{2}$