

2005 AMC10A 试题

Problem 1

While eating out, Mike and Joe each tipped their server 2 dollars. Mike tipped 10% of his bill and Joe tipped 20% of his bill. What was the difference, in dollars between their bills?

当 Mike 和 Joe 在外面吃饭时，每人都给了他们的服务员 2 美元小费。Mike 所给的小费占了他的账单的 10%，Joe 所给的小费占了他的账单的 20%。那么他们的账单相差多少美元？

- (A) 2 (B) 4 (C) 5 (D) 10 (E) 20

Problem 2

For each pair of real numbers $a \neq b$, define the operation $\star$ as

$$(a \star b) = \frac{a+b}{a-b}.$$

What is the value of $((1 \star 2) \star 3)$?

对于每对实数 $a \neq b$ ，定义操作符 $\star$ 如下：

$$(a \star b) = \frac{a+b}{a-b}.$$

那么表达式 $((1 \star 2) \star 3)$ 的值是多少？

- (A) $-\frac{2}{3}$ (B) $-\frac{1}{5}$ (C) 0 (D) $\frac{1}{2}$ (E) 这个值无定义

Problem 3

The equations $2x + 7 = 3$ and $bx - 10 = -2$ have the same solution. What is the value of b ?

方程 $2x + 7 = 3$ 和 $bx - 10 = -2$ 的根相同。那么 b 的值是多少？

- (A) -8 (B) -4 (C) 2 (D) 4 (E) 8

Problem 4

A rectangle with a diagonal of length x is twice as long as it is wide. What is the area of the rectangle?

一个长方形的长是宽的 2 倍，且对角线长度为 x 。那么此长方形的面积是多少？

- (A) $\frac{1}{4}x^2$ (B) $\frac{2}{5}x^2$ (C) $\frac{1}{2}x^2$ (D) x^2 (E) $\frac{3}{2}x^2$

Problem 5

A store normally sells windows at \$100 each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How many dollars will they save if they purchase the windows together rather than separately?

商店通常以每扇 100 美元的价格出售窗户。本周该店为客户提供买四送一的优惠。

Dave 需要七扇窗户，Doug 需要八扇。如果他们一起而不是分开购买窗户，可以节省多少美元？

- (A) 100 (B) 200 (C) 300 (D) 400 (E) 500

Problem 6

The average (mean) of 20 numbers is 30, and the average of 30 other numbers is 20. What is the average of all 50 numbers?

20 个数的平均值是 30，其他 30 个数的平均值是 20。那么所有这 50 个数的平均值是多少？

- (A) 23 (B) 24 (C) 25 (D) 26 (E) 27

Problem 7

Josh and Mike live 13 miles apart. Yesterday Josh started to ride his bicycle toward Mike's house. A little later Mike started to ride his bicycle toward Josh's house. When they met, Josh had ridden for twice the length of time as Mike and at four-fifths of Mike's rate. How many miles had Mike ridden when they met?

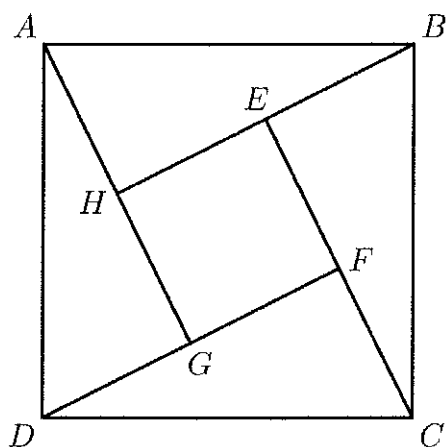
Josh 和 Mike 的家相距 13 英里。昨天 Josh 开始骑自行车去 Mike 家。过了一会儿，Mike 开始骑自行车去 Josh 的家。当他们相遇时，Josh 所花的时间是迈克的两倍，速度是 Mike 的五分之四。则当他们相遇时 Mike 已经骑了多少英里？

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Problem 8

In the figure, the length of side AB of square $ABCD$ is $\sqrt{50}$ and $BE = 1$. What is the area of the inner square $EFGH$?

如下图所示，正方形 $ABCD$ 中，边 AB 的长度是 $\sqrt{50}$ ， $BE = 1$ 。那么内部正方形 $EFGH$ 的面积是多少？



- (A) 25 (B) 32 (C) 36 (D) 40 (E) 42

Problem 9

Three tiles are marked X and two other tiles are marked O . The five tiles are randomly arranged in a row. What is the probability that the arrangement reads $XOXOX$?

三块瓷砖标有字母 X ，其他两块标有字母 O 。将这五块瓷砖随机排成一行。问排列是 $XOXOX$ 的概率是多少？

- (A) $\frac{1}{12}$ (B) $\frac{1}{10}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

Problem 10

There are two values of a for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for x . What is the sum of those values of a ?

使得方程 $4x^2 + ax + 8x + 9 = 0$ 只有一个解的 a 的值有 2 个。那么 a 的这 2 个值之和是多少?

- (A) -16 (B) -8 (C) 0 (D) 8 (E) 20

Problem 11

A wooden cube n units on a side is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is n ?

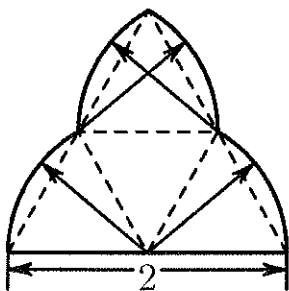
一个边长为 n 的木头正方体的六个面均被涂成红色, 然后切割为 n^3 个小的单位正方体。这些单位正方体的面的总数中, 红色的面恰好占了四分之一。问 n 是多少?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Problem 12

The figure shown is called a *trefoil* and is constructed by drawing circular sectors about the sides of the congruent equilateral triangles. What is the area of a trefoil whose horizontal base has length 2?

如图所示的图形被称为三叶形, 是通过在若干全等等边三角形的边上绘制扇形而构成。那么水平底边长度为 2 的三叶形的面积是多少?



- (A) $\frac{1}{3}\pi + \frac{\sqrt{3}}{2}$ (B) $\frac{2}{3}\pi$ (C) $\frac{2}{3}\pi + \frac{\sqrt{3}}{4}$ (D) $\frac{2}{3}\pi + \frac{\sqrt{3}}{3}$ (E) $\frac{2}{3}\pi + \frac{\sqrt{3}}{2}$

Problem 13

How many positive integers n satisfy the following condition:

$$(130n)^{50} > n^{100} > 2^{200}?$$

有多少个正整数 n 满足下面的条件:

$$(130n)^{50} > n^{100} > 2^{200}?$$

- (A) 0 (B) 7 (C) 12 (D) 65 (E) 125

Problem 14

How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

有多少个三位数具有这样的性质: 中间位的数字是第一位和最后一位数字的平均值?

- (A) 41 (B) 42 (C) 43 (D) 44 (E) 45

Problem 15

How many positive cubes divide $3! \cdot 5! \cdot 7!$?

有多少个立方数能够整除 $3! \cdot 5! \cdot 7!$?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 16

The sum of the digits of a two-digit number is subtracted from the number. The units digit of the result is 6. How many two-digit numbers have this property?

一个两位数减去这个两位数的 2 个位上数字之和, 结果的个位是 6. 有多少个两位数具有这样的性质?

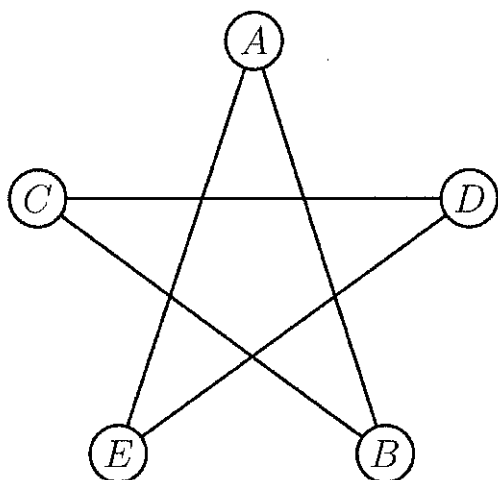
- (A) 5 (B) 7 (C) 9 (D) 10 (E) 19

Problem 17

In the five-sided star shown, the letters A , B , C , D , and E are replaced by the numbers 3, 5, 6, 7, and 9, although not necessarily in this order. The sums of the numbers at the ends of the line segments AB , BC , CD , DE , and EA form an

arithmetic sequence, although not necessarily in this order. What is the middle term of the sequence?

在如图所示的五角星中, 字母 A, B, C, D, E 被数字 3, 5, 6, 7, 9 代替(不一定按此顺序)。线段 AB, BC, CD, DE 和 EA 的两端数字之和形成一个等差数列(不一定按此顺序)。那么数列的中间项是多少?



- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

Problem 18

Team A and team B play a series. The first team to win three games wins the series. Each team is equally likely to win each game, there are no ties, and the outcomes of the individual games are independent. If team B wins the second game and team A wins the series, what is the probability that team B wins the first game?

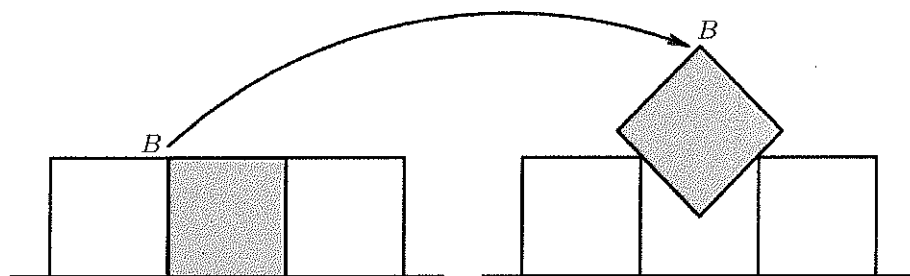
A 队和 B 队进行系列赛。第一支赢得三场比赛的球队就赢得系列赛。每支球队赢得每场比赛的可能性相同, 没有平局, 而且每场比赛的结果是独立的。如果 B 队赢得第二场比赛, A 队赢得系列赛, 则 B 队赢得第一场比赛的概率是多少?

- (A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

Problem 19

Three one-inch squares are placed with their bases on a line. The center square is lifted out and rotated 45 degrees, as shown. Then it is centered and lowered into its original location until it touches both of the adjoining squares. How many inches is the point B from the line on which the bases of the original squares were placed?

三个边长为 1 英寸的正方形的底边放置在同一条直线上。如图所示，将中间位置的正方形拿出来并旋转 45 度。然后将其居中并降低到它原来所在的位置，直到它接触到相邻的两个正方形。问点 B 和原来正方形底边所在直线的距离是多少英寸？



- (A) 1 (B) $\sqrt{2}$ (C) $\frac{3}{2}$ (D) $\sqrt{2} + \frac{1}{2}$ (E) 2

Problem 20

An equiangular octagon has four sides of length 1 and four sides of length $\frac{\sqrt{2}}{2}$, arranged so that no two consecutive sides have the same length. What is the area of the octagon?

一个等角八边形有 4 条边长度均为 1，其余四条边长度均为 $\frac{\sqrt{2}}{2}$ ，且相邻的两条边长度均不相等。那么八边形的面积是多少？

- (A) $\frac{7}{2}$ (B) $\frac{7\sqrt{2}}{2}$ (C) $\frac{5+4\sqrt{2}}{2}$ (D) $\frac{4+5\sqrt{2}}{2}$ (E) 7

Problem 21

For how many positive integers n does $1 + 2 + \dots + n$ evenly divide $6n$?

有多少个正整数 n ，使得 $1 + 2 + \dots + n$ 能够整除 $6n$ ？

- (A) 3 (B) 5 (C) 7 (D) 9 (E) 11

Problem 22

Let S be the set of the 2005 smallest positive multiples of 4, and let T be the set of the 2005 smallest positive multiples of 6. How many elements are common to S and T ?

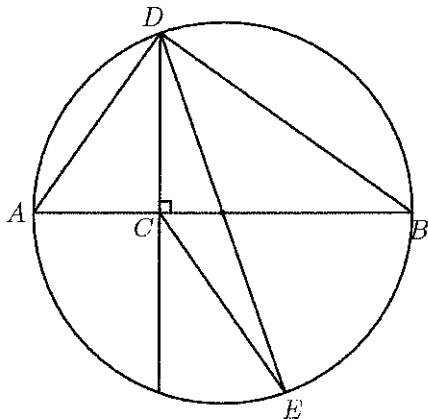
设 S 为 4 的前 2005 个最小正倍数组成的集合，设 T 为 6 的前 2005 个最小正倍数组成的集合。那么 S 和 T 有多少个公共的元素？

- (A) 166 (B) 333 (C) 500 (D) 668 (E) 1001

Problem 23

Let AB be a diameter of a circle and let C be a point on AB with $2 \cdot AC = BC$. Let D and E be points on the circle such that $DC \perp AB$ and DE is a second diameter. What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?

AB 为圆的直径， C 是 AB 上一点，满足 $2 \cdot AC = BC$ 。点 D 和 E 均为圆上的点，满足 $DC \perp AB$ ，且 DE 为第二条直径。那么 $\triangle DCE$ 的面积与 $\triangle ABD$ 的面积比值是多少？



- (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

Problem 24

For each positive integer $n > 1$, let $P(n)$ denote the greatest prime factor of n . For how many positive integers n is it true that both

$$P(n) = \sqrt{n} \text{ and } P(n+48) = \sqrt{n+48}?$$

对于每个正整数 $n > 1$, 令 $P(n)$ 表示 n 的最大质因数。那么有多少个正整数 n , 使得 $P(n) = \sqrt{n}$ 且 $P(n+48) = \sqrt{n+48}$?

- (A) 0 (B) 1 (C) 3 (D) 4 (E) 5

Problem 25

In ABC we have $AB = 25$, $BC = 39$, and $AC = 42$. Points D and E are on AB and AC respectively, with $AD = 19$ and $AE = 14$. What is the ratio of the area of triangle ADE to the area of the quadrilateral $BCED$?

在三角形 ABC 中, 我们有 $AB = 25$, $BC = 39$, $AC = 42$ 。点 D 和点 E 分别在 AB 和 AC 上, 满足 $AD = 19$, $AE = 14$ 。则三角形 ADE 的面积和四边形 $BCED$ 的面积比值是多少?

- (A) $\frac{266}{1521}$ (B) $\frac{19}{75}$ (C) $\frac{1}{3}$ (D) $\frac{19}{56}$ (E) 1