2005 AMC10B 试题

Problem 1

A scout troop buys 1000 candy bars at a price of five for \$2. They sell all the candy bars at a price of two for \$1. What was the profit, in dollars?

一支童子军以 5 个糖果 2 美元的价格购买了 1000 个糖果。他们以 2 个糖果 1 美元的价格出售所有的糖果。则利润是多少美元?

(A)100 (B)200 (C)300 (D)400 (E)500

Problem 2

A positive number x has the property that $x\%$ of x is 4. What is x?

一个正数x具有这样的性质：x的$x\%$等于 4.则x是多少?

(A)2 (B)4 (C)10 (D)20 (E)40

Problem 3

A gallon of paint is used to paint a room. One third of the paint is used on the first day. On the second day, one third of the remaining paint is used. What fraction of the original amount of paint is available to use on the third day?

使用一加仑油漆粉刷房间。第一天用了油漆的三分之一。第二天用了剩下油漆的三分之一。问第三天还可以使用原来油漆总量的几分之几?

(A)$\frac{1}{10}$ (B)$\frac{1}{9}$ (C)$\frac{1}{3}$ (D)$\frac{4}{9}$ (E)$\frac{5}{9}$

Problem 4

For real numbers a and b, define $a \diamond b = \sqrt{a^2 + b^2}$. What is the value of

$(5 \diamond 12) \diamond ((-12) \diamond (-5))$?

a 和 b均为实数，定义$a \diamond b = \sqrt{a^2 + b^2}$。那么表达式

$(5 \diamond 12) \diamond ((-12) \diamond (-5))$的值是多少?

(A)0 (B)$\frac{17}{2}$ (C)13 (D)$13\sqrt{2}$ (E)26

Problem 5

Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fifth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

Brianna 用她周末工作挣的一部分钱买了几张价格相等的 CD。她用五分之一的钱买了三分之一的 CD。当她买了所有的 CD 后，她还剩几分之几的钱?

(A)$\frac{1}{5}$ (B)$\frac{1}{3}$ (C)$\frac{2}{5}$ (D)$\frac{2}{3}$ (E)$\frac{4}{5}$

Problem 6

At the beginning of the school year, Lisa's goal was to earn an A on at least 80% of her 50 quizzes for the year. She earned an A on 22 of the first 30 quizzes. If she is to achieve her goal, on at most how many of the remaining quizzes can she earn a grade lower than an A?

在学年开始时，Lisa 的目标是在今年的 50 次测验中，至少 80%的测验获得 A。她在前 30 次测验中，有 22 次测验获得了 A。如果她想实现自己的目标，那么剩下的测验中，她最多有多少次测验允许获得低于 A 的分数?

(A)1 (B)2 (C)3 (D)4 (E)5

Problem 7

A circle is inscribed in a square, then a square is inscribed in this circle, and finally, a circle is inscribed in this square. What is the ratio of the area of the smaller circle to the area of the larger square?

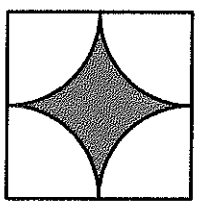
一个圆内切于一个正方形中，然后这个圆外接于一个正方形，最后，一个圆又内切于这个正方形。那么较小的圆的面积和较大的正方形的面积的比值是多少?

(A)$\frac{\pi}{16}$ (B)$\frac{\pi}{8}$ (C)$\frac{3\pi}{16}$ (D)$\frac{\pi}{4}$ (E)$\frac{\pi}{2}$

Problem 8

An 8-foot by 10-foot floor is tiled with square tiles of size 1 foot by 1 foot. Each tile has a pattern consisting of four white quarter circles of radius $1/2$ foot centered at each corner of the tile. The remaining portion of the tile is shaded. How many square feet of the floor are shaded?

一块 8 英尺×10 英尺的地板是由尺寸为 1 英尺×1 英尺的方形瓷砖铺成的。每块瓷砖上都有这样一个图案：4 个白色的四分之一圆的半径均为$1/2$英尺，且圆心位于瓷砖的每个顶点，而瓷砖的剩余部分则涂成阴影。那么地板有多少平方英尺的面积被涂成阴影?



(A) $80-20\pi$ (B) $60-10\pi$ (C) $80-10\pi$ (D) $60+10\pi$ (E) $80+10\pi$

Problem 9

One fair die has faces 1, 1, 2, 2, 3, 3 and another has faces 4, 4, 5, 5, 6, 6. The dice are rolled and the numbers on the top faces are added. What is the probability that the sum will be odd?

一个标准的骰子各个面上数字是 1, 1, 2, 2, 3, 3，另一个骰子各个面上数字是 4, 4, 5, 5, 6, 6. 同时投掷这 2 枚骰子，并将顶面的两个数相加。问和为奇数的概率是多少?

(A)$\frac{1}{3}$ (B)$\frac{4}{9}$ (C)$\frac{1}{2}$ (D)$\frac{5}{9}$ (E)$\frac{2}{3}$

Problem 10

In $\triangle ABC$, we have $AC=BC=7$ and $AB=2$. Suppose that D is a point on line AB such that B lies between A and D and $CD=8$. What is BD?

在$\triangle ABC$中，我们有$AC=BC=7$，$AB=2$。假设 D 是直线 AB 上一点，满足 B 位于 A 和 D 之间，且$CD=8$。则BD是多长?

(A)3 (B)$2\sqrt{3}$ (C)4 (D)5 (E)$4\sqrt{2}$

Problem 11

The first term of a sequence is 2005. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2005^{th} term of the sequence?

一个数列的第一项是 2005. 后面的每一项都是前一项各个位上数字的立方和。那么这个数列的第 2005 项是多少?

(A)29 (B)55 (C)85 (D)133 (E)250

Problem 12

Twelve fair dice are rolled. What is the probability that the product of the numbers on the top faces is prime?

掷了 12 枚标准的骰子。则顶面所有数字之积是一个质数的概率是多少?

(A) $\left(\frac{1}{12}\right)^{12}$ (B) $\left(\frac{1}{6}\right)^{12}$ (C)2 $\left(\frac{1}{6}\right)^{11}$ (D)$\frac{5}{2}\left(\frac{1}{6}\right)^{11}$ (E) $\left(\frac{1}{6}\right)^{10}$

Problem 13

How many numbers between 1 and 2005 are integer multiples of 3 or 4 but not 12?

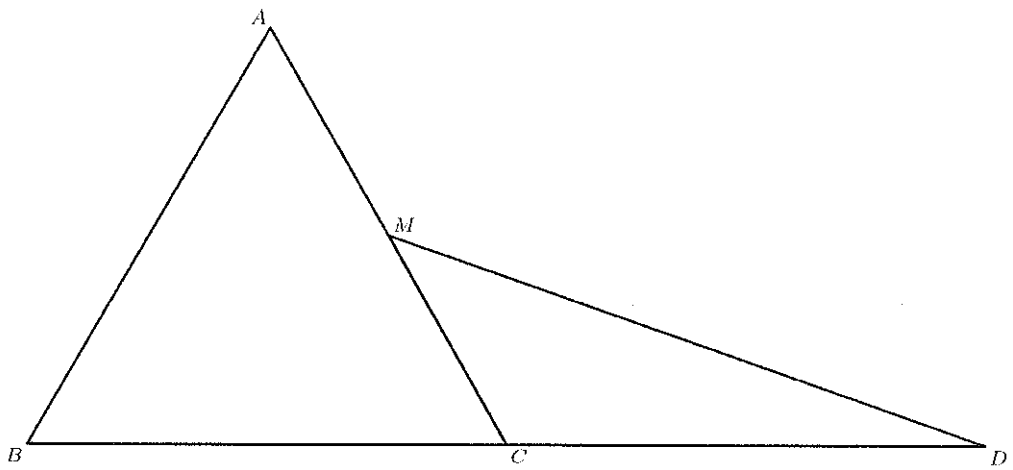
1 到 2005 之间有多少数是 3 或 4 的整数倍，但不是 12 的整数倍?

(A)501 (B)668 (C)835 (D)1002 (E)1169

Problem 14

Equilateral $\triangle ABC$ has side length 2, M is the midpoint of $\overline{AC}$, and C is the midpoint of $\overline{BD}$. What is the area of $\triangle CDM$?

等边$\triangle ABC$的边长是 2，M 是$\overline{AC}$的中点，C 是$\overline{BD}$的中点。那么$\triangle CDM$的面积是多少?



(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{3}{4}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1 (E) $\sqrt{2}$

Problem 15

An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is \$20 or more?

一个信封里有八张钞票：2 张一元、2 张五元、2 张十元和 2 张二十元。从中不放回地随机抽取两张钞票。问它们的总额等于或大于 20 美元的概率是多少？

(A)$\frac{1}{4}$ (B)$\frac{2}{5}$ (C)$\frac{3}{7}$ (D)$\frac{1}{2}$ (E)$\frac{2}{3}$

Problem 16,

The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $x^2 + px + m = 0$, and none of m, n, and p is zero. What is the value of n/p?

一元二次方程$x^2 + mx + n = 0$的两个根分别是$x^2 + px + m = 0$的两个根的 2 倍，并且 $m, n,$ 和 p 均不为 0.那么n/p是多少？

(A)1 (B)2 (C)4 (D)8 (E)16

Problem 17

Suppose that $4^a = 5$, $5^b = 6$, $6^c = 7$, and $7^d = 8$. What is $a \cdot b \cdot c \cdot d$?

假设 $4^a = 5$, $5^b = 6$, $6^c = 7$, $7^d = 8$，则$a \cdot b \cdot c \cdot d$是多少？

(A)1 (B)$\frac{3}{2}$ (C)2 (D)$\frac{5}{2}$ (E)3

Problem 18

All of David's telephone numbers have the form $555 - abc - defg$, where a, b, c, d, e, f, and g are distinct digits and in increasing order, and none is either 0 or 1. How many different telephone numbers can David have?

David 的所有电话号码都是这种形式：$555 - abc - defg$，其中 $a, b, c, d, e, f,$ 和 g是不同的数字，以升序排列，且每个数字都不是 0 或 1.那么 David 一共有多少个不同的电话号码?

(A)1 (B)2 (C)7 (D)8 (E)9

Problem 19

On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

在某次数学考试中，10%的学生得了 70 分，25%的学生得了 80 分，20%的学生 85 分，15%的学生得了 90 分，其余人得了 95 分。那么这次考试的平均值和中位数的差是多少?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 5

Problem 20

What is the average (mean) of all 5-digit numbers that can be formed by using each of the digits 1, 3, 5, 7, and 8 exactly once?

使用 1, 3, 5, 7,和 8中的每个数字各一次，组成的所有 5 位数的平均值是多少?

(A)48000 (B)49999.5 (C)53332.8 (D)55555 (E)56432.8

Problem 21

Forty slips are placed into a hat, each bearing a number 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10, with each number entered on four slips. Four slips are drawn from the hat at random and without replacement. Let P be the probability that all four slips bear the same

number. Let q be the probability that two of the slips bear a number a and the other two bear a number $b \neq a$. What is the value of $\frac{q}{p}$?

四十张纸条放在一顶帽子里，每张纸条上都有 1, 2, 3, 4, 5, 6, 7, 8, 9, 或 10其中一个数字，每个数字都写在四张纸条上。从帽子里不放回地随机抽出四张纸条。令 p表示这四张纸条上的数字都相同的概率。令 q 表示其中两张纸条上都是数字a，另外两张纸条上都是数字$b \neq a$的概率。那么$\frac{q}{p}$的值是多少？

(A)162 (B)180 (C)324 (D)360 (E)720

Problem 22

For how many positive integers n less than or equal to 24 is $n!$ evenly divisible by $1 + 2 + \ldots + n$?

有多少个小于或等于 24 的正整数n，满足$n!$能被$1 + 2 + \ldots + n$整除?

(A)8 (B)12 (C)16 (D)17 (E)21

Problem 23

In trapezoid $ABCD$ we have $\overline{AB}$ parallel to $\overline{DC}$, E as the midpoint of $\overline{BC}$, and F as the midpoint of $\overline{DA}$. The area of $ABEF$ is twice the area of $FECD$. What is AB/DC?

在梯形 ABCD 中，我们有$\overline{AB}$平行于$\overline{DC}$，E 是$\overline{BC}$的中点，F 是$\overline{DA}$的中点。$ABEF$的面积是$FECD$面积的 2 倍。则AB/DC是多少?

(A)2 (B)3 (C)5 (D)6 (E)8

Problem 24

Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m. What is $x + y + m$?

令 x 和 y 均为两位整数，满足 y 是通过将 x 的各个位上数字反序排列得到的。整数 x 和 y 满足 $x^2 - y^2 = m^2$，这里 m 是某个正整数。那么 $x + y + m$ 是多少？

(A)88 (B)112 (C)116 (D)144 (E)154

Problem 25

A subset B of the set of integers from 1 to 100, inclusive, has the property that no two elements of B sum to 125. What is the maximum possible number of elements in B?

包含 1 到 100 之间（包括 1 和 100）所有整数的集合的某个子集 B 具有这样的性质：B 中不存在和为 125 的两个元素。问 B 中元素个数的最大可能值是多少？

(A)50 (B)51 (C)62 (D)65 (E)68