

2006 AMC10A 试题

Problem 1

Sandwiches at Joe's Fast Food cost \$3 each and sodas cost \$2 each. How many dollars will it cost to purchase 5 sandwiches and 8 sodas?

Joe 快餐店的三明治每份 3 美元，苏打水每杯 2 美元。那么买 5 份三明治和 8 杯苏打水要多少美元？

- (A) 31 (B) 32 (C) 33 (D) 34 (E) 35

Problem 2

Define $x \otimes y = x^3 - y$. What is $h \otimes (h \otimes h)$?

定义 $x \otimes y = x^3 - y$ ，则 $h \otimes (h \otimes h)$ 是多少？

- (A) $-h$ (B) 0 (C) h (D) $2h$ (E) h^3

Problem 3

The ratio of Mary's age to Alice's age is 3 : 5. Alice is 30 years old. How many years old is Mary?

Mary 和 Alice 的年龄的比值是 3 : 5。已知 Alice 是 30 岁，那么 Mary 是多少岁？

- (A) 15 (B) 18 (C) 20 (D) 24 (E) 50

Problem 4

A digital watch displays hours and minutes with AM and PM. What is the largest possible sum of the digits in the display?

一个电子手表以上午 (AM) 和下午 (PM) 来显示小时和分钟 (例如: 15:30 则显示为下午 3:30)。那么所显示的数的各个位上数字之和的最大可能值是多少？

- (A) 17 (B) 19 (C) 21 (D) 22 (E) 23

Problem 5

Doug and Dave shared a pizza with 8 equally-sized slices. Doug wanted a plain pizza, but Dave wanted anchovies on half of the pizza. The cost of a plain pizza was 8 dollars, and there was an additional cost of 2 dollars for putting anchovies on one half. Dave ate all of the slices of anchovy pizza and one plain slice. Doug ate the remainder. Each then paid for what he had eaten. How many more dollars did Dave pay than Doug?

Doug 和 Dave 分享了一份披萨，里面有 8 片大小相等的披萨。Doug 想要一个原味披萨，但 Dave 想要披萨上有一半是凤尾鱼。一个普通披萨的价格是 8 美元，而另一半上放凤尾鱼则需要额外支付 2 美元。Dave 吃了所有的凤尾鱼披萨片和一片普通披萨片。Doug 把剩下的都吃了。然后每个人都付了各自所吃的披萨的钱。那么 Dave 比 Doug 多付了多少美元？

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 6

What non-zero real value for x satisfies $(7x)^{14} = (14x)^7$?

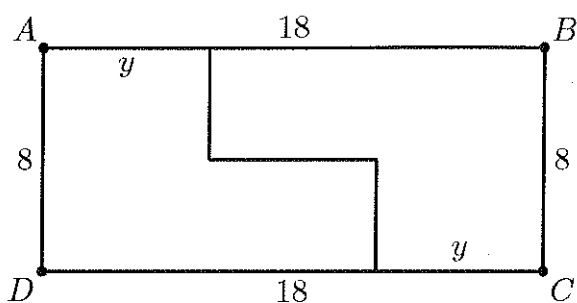
x 是什么非零实数时，满足方程 $(7x)^{14} = (14x)^7$?

- (A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) 1 (D) 7 (E) 14

Problem 7

The 8×18 rectangle $ABCD$ is cut into two congruent hexagons, as shown, in such a way that the two hexagons can be repositioned without overlap to form a square. What is y ?

尺寸为 8×18 的矩形 $ABCD$ 被分成 2 个全等的六边形，如图所示，且这 2 个六边形在不重叠的情况下可以重新拼成一个正方形。则 y 是多少？



- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Problem 8

A parabola with equation $y = x^2 + bx + c$ passes through the points $(2, 3)$ and $(4, 3)$. What is c ?

一个方程为 $y = x^2 + bx + c$ 的抛物线通过点 $(2, 3)$ 和 $(4, 3)$, 则 c 是多少?

- (A) 2 (B) 5 (C) 7 (D) 10 (E) 11

Problem 9

How many sets of two or more consecutive positive integers have a sum of 15?

有多少个由两个或多个连续的正整数组成的集合的所有元素之和为 15?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 10

For how many real values of x is $\sqrt{120 - \sqrt{x}}$ an integer?

有多少个实数 x , 使得 $\sqrt{120 - \sqrt{x}}$ 是个整数?

- (A) 3 (B) 6 (C) 9 (D) 10 (E) 11

Problem 11

Which of the following describes the graph of the equation $(x + y)^2 = x^2 + y^2$?

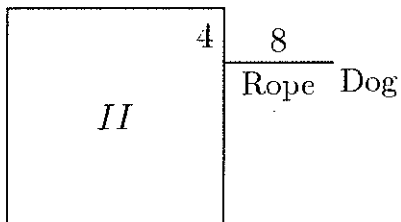
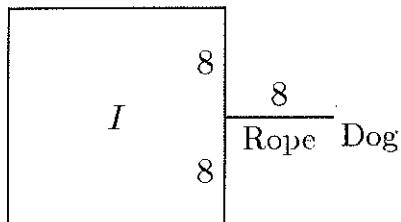
下面哪个描述了方程 $(x + y)^2 = x^2 + y^2$ 的图像?

- (A) the empty set (B) one point (C) two lines (D) a circle (E) the entire plane

(A) 空集 (B) 一个点 (C) 两条直线 (D) 一个圆 (E) 整个平面

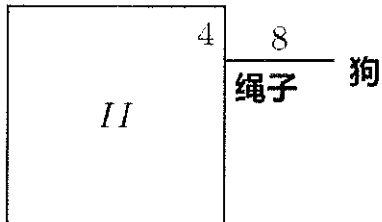
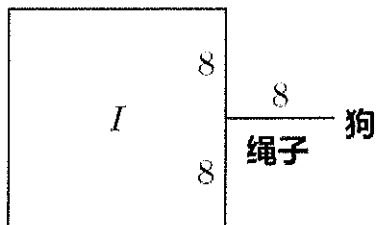
Problem 12

Rolly wishes to secure his dog with an 8-foot rope to a square shed that is 16 feet on each side. His preliminary drawings are shown.



Which of these arrangements give the dog the greater area to roam, and by how many square feet?

Rolly 希望用一根 8 英尺长的绳子把他的狗拴在一个边长为 16 英尺的正方形棚子里。他的初步图纸如图所示。



以下哪种安排让狗拥有更大的漫游面积，大多少平方英尺？

(A) I, by 8π (B) I, by 6π (C) II, by 4π (D) II, by 8π (E) II, by 10π

(A) I, 大 8π (B) I, 大 6π (C) II, 大 4π (D) II, 大 8π (E) II, 大 10π

Problem 13

A player pays \$5 to play a game. A die is rolled. If the number on the die is odd, the game is lost. If the number on the die is even, the die is rolled again. In this case the player wins if the second number matches the first and loses otherwise. How much should the player win if the game is fair? (In a fair game the probability of winning times the amount won is what the player should pay.)

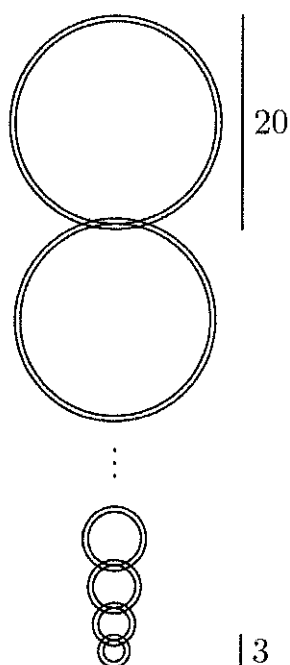
一个玩家付费 5 美元玩游戏。首先掷一枚骰子。如果骰子上的数是奇数，游戏就输了。如果骰子上的数为偶数，则再次掷这枚骰子。在这种情况下，如果第二个数与第一个数匹配，那么玩家将获胜，否则就输了。如果游戏公平，玩家应该赢多少美元？（在公平的游戏里，获胜的概率乘以获胜的金额是玩家应该支付的金额。）

(A) 12 (B) 30 (C) 50 (D) 60 (E) 100

Problem 14

A number of linked rings, each 1 cm thick, are hanging on a peg. The top ring has an outside diameter of 20 cm. The outside diameter of each of the other rings is 1 cm less than that of the ring above it. The bottom ring has an outside diameter of 3 cm. What is the distance, in cm, from the top of the top ring to the bottom of the bottom ring?

有许多相连的环，每一个都是 1 厘米厚，挂在一个钉子上。顶环的外径为 20 厘米。其他每个环的外径比其上方的环的外径小 1 厘米。底环的外径为 3 厘米。从顶环顶部到底环底部的距离是多少厘米？



- (A) 171 (B) 173 (C) 182 (D) 188 (E) 210

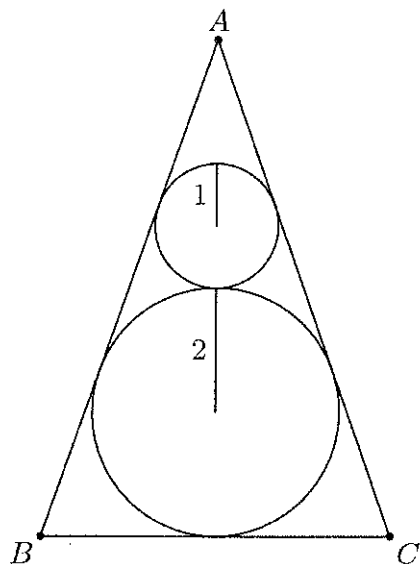
Problem 15

Odell and Kershaw run for 30 minutes on a circular track. Odell runs clockwise at 250 m/min and uses the inner lane with a radius of 50 meters. Kershaw runs counterclockwise at 300 m/min and uses the outer lane with a radius of 60 meters, starting on the same radial line as Odell. How many times after the start do they pass each other?

Odell 和 Kershaw 在环形跑道上跑了 30 分钟。Odell 以 250 米/分钟的速度顺时针跑步，使用半径为 50 米的内跑道。Kershaw 以 300 米/分钟的速度逆时针跑步，使用半径为 60 米的外车道，从与 Odell 相同的径向线开始。在他们起跑后，他们一共相遇了多少次？

- (A) 29 (B) 42 (C) 45 (D) 47 (E) 50

Problem 16



A circle of radius 1 is tangent to a circle of radius 2. The sides of $\triangle ABC$ are tangent to the circles as shown, and the sides $\overline{AB}$ and $\overline{AC}$ are congruent. What is the area of $\triangle ABC$?

一个半径为 1 的圆和一个半径为 2 的圆相切。 $\triangle ABC$ 的各边和两圆相切，如图所示，且边 $\overline{AB}$ 等于边 $\overline{AC}$ 。那么 $\triangle ABC$ 的面积是多少？

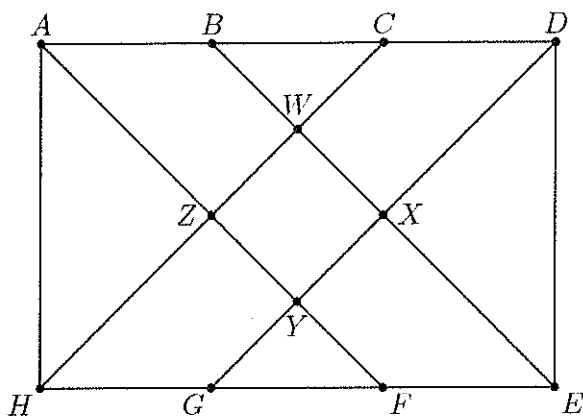
- (A) $\frac{35}{2}$ (B) $15\sqrt{2}$ (C) $\frac{64}{3}$ (D) $16\sqrt{2}$ (E) 24

Problem 17

In rectangle $ADEH$, points B and C trisect $\overline{AD}$, and points G and F trisect $\overline{HE}$. In addition, $AH = AC = 2$, and $AD = 3$. What is the area of quadrilateral $WXYZ$ shown in the figure?

在矩形 $ADEH$ 中，点 B 和 C 三等份 $\overline{AD}$ ，点 G 和 F 三等份 $\overline{HE}$ 。另外，

$AH = AC = 2$ ， $AD = 3$ 。那么如图所示的四边形 $WXYZ$ 的面积是多少？



- (A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{2}$ (E) $\frac{2\sqrt{3}}{3}$

Problem 18

A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?

一个车牌在某种特定状态下由4位不一定不同的数字和2个也不一定不同的字母组成。这六个字符可以以任何顺序出现，但这两个字母必须相邻出现。那么这样不同的车牌一共有多少个？

- (A) $10^4 \times 26^2$ (B) $10^3 \times 26^3$ (C) $5 \times 10^4 \times 26^2$ (D) $10^2 \times 26^4$ (E) $5 \times 10^3 \times 26^3$

Problem 19

How many non-similar triangles have angles whose degree measures are distinct positive integers in arithmetic progression?

有多少个不相似的三角形，其内角度数是成等差数列的不同的正整数？

- (A) 0 (B) 1 (C) 59 (D) 89 (E) 178

Problem 20

Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?

从 1 到 2006 (包括 1 和 2006) 之间随机选择 6 个不同的正整数。则这些整数中存在某一对整数, 它们的差是 5 的倍数的概率是多少?

- (A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{4}{5}$ (E) 1

Problem 21

How many four-digit positive integers have at least one digit that is a 2 or a 3?

有多少个 4 位的正整数至少有一个数字是 2 或 3?

- (A) 2439 (B) 4096 (C) 4903 (D) 4904 (E) 5416

Problem 22

Two farmers agree that pigs are worth \$300 and that goats are worth \$210. When one farmer owes the other money, he pays the debt in pigs or goats, with "change" received in the form of goats or pigs as necessary. (For example, a \$390 debt could be paid with two pigs, with one goat received in change.) What is the amount of the smallest positive debt that can be resolved in this way?

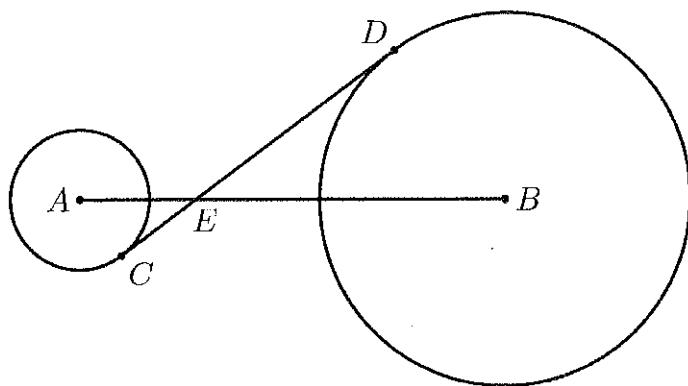
两位农民商定, 一头猪价值 300 美元, 一头山羊价值 210 美元。当一个农民欠另一农民一笔钱时, 他可以用猪或山羊偿还债务, 而必要时找的“零钱”也是山羊或猪。(例如, 一笔 390 美元的债务可以用两头猪来偿还, 而找的零钱则是一头山羊。) 用这种方法可以解决的最小正债务是多少美元?

- (A) 5 (B) 10 (C) 30 (D) 90 (E) 210

Problem 23

Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent intersects the circles at C and D , respectively. Lines AB and CD intersect at E , and $AE = 5$. What is CD ?

圆心为 A 和 B 的两个圆, 半径分别为 3 和 8. 一条内公切线分别交两圆于 C 和 D . 直线 AB 和 CD 交于 E , 且 $AE = 5$. 则 CD 是多少?



- (A) 13 (B) $\frac{44}{3}$ (C) $\sqrt{221}$ (D) $\sqrt{255}$ (E) $\frac{55}{3}$

Problem 24

Centers of adjacent faces of a unit cube are joined to form a regular octahedron. What is the volume of this octahedron?

将一个单位正方体的每个相邻面的中心连接起来，形成一个正八面体。那么这个八面体的体积是多少？

- (A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

Problem 25

A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

一只虫子从立方体的一个顶点开始，并根据以下规则沿着立方体的边移动。在每个顶点，虫子会选择沿着该顶点发出的三条边中的一条移动。每条边被选择的概率相等，所有选择都是独立的。七次移动后，这只虫子恰好访问过每一个顶点的概率是多少？

- (A) $\frac{1}{2187}$ (B) $\frac{1}{729}$ (C) $\frac{2}{243}$ (D) $\frac{1}{81}$ (E) $\frac{5}{243}$