

2006 AMC10B 试题

Problem 1

What is $(-1)^1 + (-1)^2 + \dots + (-1)^{2006}$?

$(-1)^1 + (-1)^2 + \dots + (-1)^{2006}$ 的值是多少?

- (A) -2006 (B) -1 (C) 0 (D) 1 (E) 2006

Problem 2

For real numbers x and y , define $x \spadesuit y = (x + y)(x - y)$. What is $3 \spadesuit (4 \spadesuit 5)$?

对于实数 x 和 y , 定义 $x \spadesuit y = (x + y)(x - y)$, 则 $3 \spadesuit (4 \spadesuit 5)$ 是多少?

- (A) -72 (B) -27 (C) -24 (D) 24 (E) 72

Problem 3

A football game was played between two teams, the Cougars and the Panthers. The two teams scored a total of 34 points, and the Cougars won by a margin of 14 points. How many points did the Panthers score?

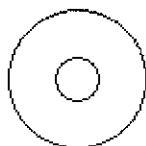
美洲狮队和黑豹队进行了一场足球赛。两队共得 34 分, 美洲狮队以 14 分的优势获胜。黑豹队得了多少分?

- (A) 10 (B) 14 (C) 17 (D) 20 (E) 24

Problem 4

Circles of diameter 1 inch and 3 inches have the same center. The smaller circle is painted red, and the portion outside the smaller circle and inside the larger circle is painted blue. What is the ratio of the blue-painted area to the red-painted area?

直径为 1 英寸和 3 英寸的两个圆为同心圆。小圆被涂成红色, 在小圆外但在大圆内的部分被涂成蓝色。那么蓝色区域的面积和红色区域的面积的比值是多少?



- (A) 2 (B) 3 (C) 6 (D) 8 (E) 9

Problem 5

A 2×3 rectangle and a 3×4 rectangle are contained within a square without overlapping at any point, and the sides of the square are parallel to the sides of the two given rectangles. What is the smallest possible area of the square?

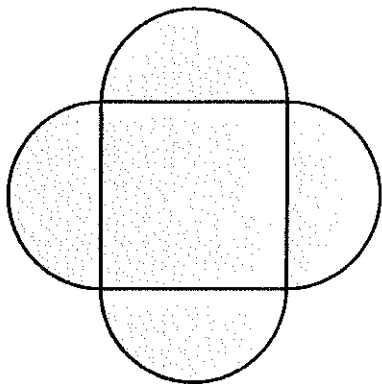
一个正方形内有一个 2×3 的矩形和一个 3×4 的矩形, 且这两个矩形不在任何点重合。正方形的边和这 2 个给定的矩形的边平行。那么这个正方形的面积的最小值是多少?

- (A) 16 (B) 25 (C) 36 (D) 49 (E) 64

Problem 6

A region is bounded by semicircular arcs constructed on the side of a square whose sides measure $\frac{2}{\pi}$, as shown. What is the perimeter of this region?

在边长为 $\frac{2}{\pi}$ 的正方形的每条边上作一个半圆, 围成图示的区域。问这个区域的周长是多少?



- (A) $\frac{4}{\pi}$ (B) 2 (C) $\frac{8}{\pi}$ (D) 4 (E) $\frac{16}{\pi}$

Problem 7

Which of the following is equivalent to $\sqrt{1 - \frac{x-1}{x}}$ when $x < 0$?

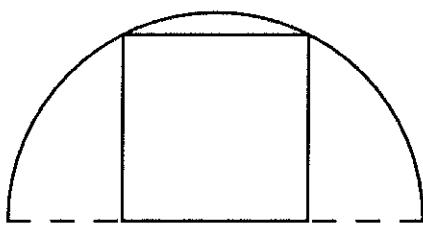
当 $x < 0$ 时, 下面哪个等于 $\sqrt{\frac{x}{1 - \frac{x-1}{x}}}$?

- (A) $-x$ (B) x (C) 1 (D) $\sqrt{\frac{x}{2}}$ (E) $x\sqrt{-1}$

Problem 8

A square of area 40 is inscribed in a semicircle as shown. What is the area of the semicircle?

一个面积为 40 的正方形内接在如图所示的半圆中。则此半圆的面积是多少?



- (A) 20π (B) 25π (C) 30π (D) 40π (E) 50π

Problem 9

Francesca uses 100 grams of lemon juice, 100 grams of sugar, and 400 grams of water to make lemonade. There are 25 calories in 100 grams of lemon juice and 386 calories in 100 grams of sugar. Water contains no calories. How many calories are in 200 grams of her lemonade?

Francesca 用 100 克柠檬汁、100 克糖和 400 克水来制作柠檬水。100 克柠檬汁含有 25 卡路里, 100 克糖含有 386 卡路里。水不含卡路里。200 克柠檬水含有多少卡路里?

- (A) 129 (B) 137 (C) 174 (D) 233 (E) 411

Problem 10

In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 15. What is the greatest possible perimeter of the triangle?

一个三角形中，各边长均为整数，其中一条边是第二条边的3倍长，第三条边的长度为15。那么这个三角形周长的最大值是多少？

- (A) 43 (B) 44 (C) 45 (D) 46 (E) 47

Problem 11

What is the tens digit in the sum $7! + 8! + 9! + \dots + 2006!$

$7! + 8! + 9! + \dots + 2006!$ 的十位上的数字是多少？

- (A) 1 (B) 3 (C) 4 (D) 6 (E) 9

Problem 12

The lines $x = \frac{1}{4}y + a$ and $y = \frac{1}{4}x + b$ intersect at the point $(1, 2)$. What is $a + b$?

直线 $x = \frac{1}{4}y + a$ 和 $y = \frac{1}{4}x + b$ 交于点 $(1, 2)$ 。那么 $a + b$ 是多少？

- (A) 0 (B) $\frac{3}{4}$ (C) 1 (D) 2 (E) $\frac{9}{4}$

Problem 13

Joe and JoAnn each bought 12 ounces of coffee in a 16 ounce cup. Joe drank 2 ounces of his coffee and then added 2 ounces of cream. JoAnn added 2 ounces of cream, stirred the coffee well, and then drank 2 ounces. What is the resulting ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee?

Joe 和 JoAnn 每人买了 12 盎司的咖啡，都分别装在一个 16 盎司的杯子里。Joe 喝了 2 盎司咖啡，然后加了 2 盎司奶油。JoAnn 加了 2 盎司奶油，搅拌好咖啡，然后喝了 2 盎司。Jo 咖啡中的奶油量与 JoAnn 咖啡中的奶油量之比是多少？

- (A) $\frac{6}{7}$ (B) $\frac{13}{14}$ (C) 1 (D) $\frac{14}{13}$ (E) $\frac{7}{6}$

Problem 14

Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + (1/b)$ and $b + (1/a)$ are the roots of the equation $x^2 - px + q = 0$. What is q ?

令 a 和 b 为方程 $x^2 - mx + 2 = 0$ 的根。假设 $a + (1/b)$ 和 $b + (1/a)$ 是方程 $x^2 - px + q = 0$ 的根，那么 q 是多少？

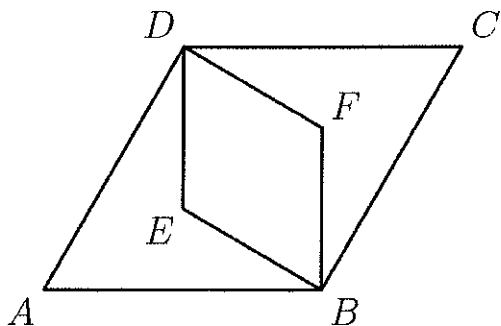
- (A) $\frac{5}{2}$ (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) 8

Problem 15

Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 24 and $\angle BAD = 60^\circ$. What is the area of rhombus $BFDE$?

菱形 $ABCD$ 相似于菱形 $BFDE$ 。菱形 $ABCD$ 的面积是 24，且 $\angle BAD = 60^\circ$ 。

那么菱形 $BFDE$ 的面积是多少？



- (A) 6 (B) $4\sqrt{3}$ (C) 8 (D) 9 (E) $6\sqrt{3}$

Problem 16

Leap Day, February 29, 2004, occurred on a Sunday. On what day of the week will Leap Day, February 29, 2020, occur?

2004 年 2 月 29 日这个闰日是个星期天。2020 年 2 月 29 日这个闰日将发生在一周中的哪一天？

- (A) 周二 (B) 周三 (C) 周四 (D) 周五 (E) 周六

Problem 17

Bob and Alice each have a bag that contains one ball of each of the colors blue, green, orange, red, and violet. Alice randomly selects one ball from her bag and puts it into Bob's bag. Bob then randomly selects one ball from his bag and puts it into Alice's bag. What is the probability that after this process the contents of the two bags are the same?

Bob 和 Alice 每人都有一个包，包里都装着蓝色、绿色、橙色、红色和紫色的球各一个。Alice 从包里随机挑选一个球，放进 Bob 的包里。然后 Bob 从他的包里随机挑选一个球，放进 Alice 的包里。经过这个过程后，两个包里装的东西一模一样的概率是多少？

- (A) $\frac{1}{10}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

Problem 18

Let $a_1, a_2, \dots$ be a sequence for which $a_1 = 2$, $a_2 = 3$, and $a_n = \frac{a_{n-1}}{a_{n-2}}$ for each positive integer $n \geq 3$. What is a_{2006} ?

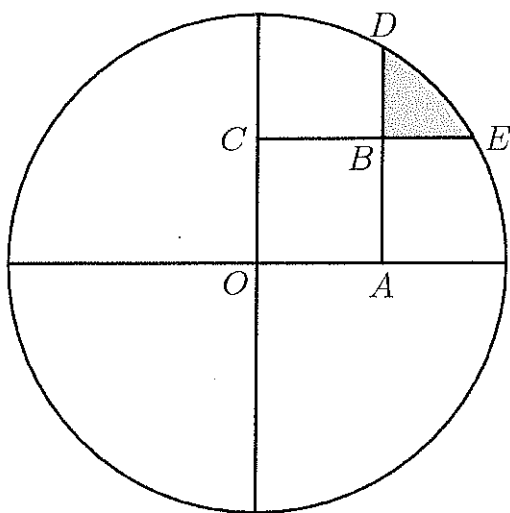
$a_1, a_2, \dots$ 是个数列，满足 $a_1 = 2, a_2 = 3$ ，且 $a_n = \frac{a_{n-1}}{a_{n-2}}$ ，对于任何正整数 $n \geq 3$ 都成立。那么 a_{2006} 是多少？

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

Problem 19

A circle of radius 2 is centered at O . Square $OABC$ has side length 1. Sides AB and CB are extended past B to meet the circle at D and E , respectively. What is the area of the shaded region in the figure, which is bounded by BD , BE , and the minor arc connecting D and E ?

一个半径为 2 的圆的圆心在 O 点。正方形 $OABC$ 的边长为 1。延长 CB 和 AB 分别交圆于 E 和 D 。则图中由 BD ， BE 和连接 D 、 E 的劣弧所包围的阴影部分区域的面积是多少？



- (A) $\frac{\pi}{3} + 1 - \sqrt{3}$ (B) $\frac{\pi}{2}(2 - \sqrt{3})$ (C) $\pi(2 - \sqrt{3})$ (D) $\frac{\pi}{6} + \frac{\sqrt{3} + 1}{2}$ (E) $\frac{\pi}{3} - 1 + \sqrt{3}$

Problem 20

In rectangle $ABCD$, we have $A = (6, -22)$, $B = (2006, 178)$, $D = (8, y)$, for some integer y . What is the area of rectangle $ABCD$?

在矩形 $ABCD$ 中，我们有 $A = (6, -22)$, $B = (2006, 178)$, $D = (8, y)$ ，其中 y 是某个整数。那么矩形 $ABCD$ 的面积是多少？

- (A) 4000 (B) 4040 (C) 4400 (D) 40,000 (E) 40,400

Problem 21

For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6, on each die are in the ratio $1 : 2 : 3 : 4 : 5 : 6$. What is the probability of rolling a total of 7 on the two dice?

有一对特殊的骰子，每个骰子投到数字 1, 2, 3, 4, 5, 或 6 的概率之比

是 $1 : 2 : 3 : 4 : 5 : 6$ 。那么这 2 个骰子投到的数字总和是 7 的概率是多少？

- (A) $\frac{4}{63}$ (B) $\frac{1}{8}$ (C) $\frac{8}{63}$ (D) $\frac{1}{6}$ (E) $\frac{2}{7}$

Problem 22

Elmo makes N sandwiches for a fundraiser. For each sandwich he uses B globs of peanut butter at 4¢ per glob and J blobs of jam at 5¢ per blob. The cost of the peanut

butter and jam to make all the sandwiches is \$2.53. Assume that B , J , and N are positive integers with $N > 1$. What is the cost of the jam Elmo uses to make the sandwiches?

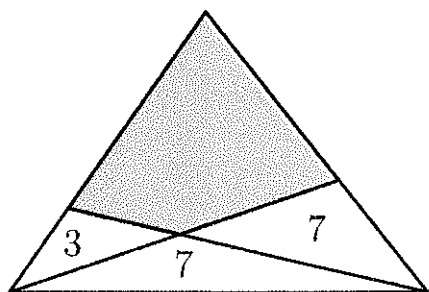
Elmo 为筹款做了 N 个三明治。对于每一个三明治，他使用 B 勺花生酱，每勺成本是 4 分，且使用 J 勺果酱，每勺果酱的成本是 5 分。制作所有三明治所需的花生酱和果酱的成本总共为 2.53 元。假设 B , J , 和 N 都是正整数，且 $N > 1$ 。则 Elmo 用来做所有三明治所需的果酱的成本是多少美元？

- (A) 1.05 (B) 1.25 (C) 1.45 (D) 1.65 (E) 1.85

Problem 23

A triangle is partitioned into three triangles and a quadrilateral by drawing two lines from vertices to their opposite sides. The areas of the three triangles are 3, 7, and 7 as shown. What is the area of the shaded quadrilateral?

在一个三角形内，通过从两个顶点到各自对边分别绘制两条线，将三角形分为三个三角形和一个四边形。三个三角形的面积分别为 3、7 和 7，如图所示。阴影四边形的面积是多少？

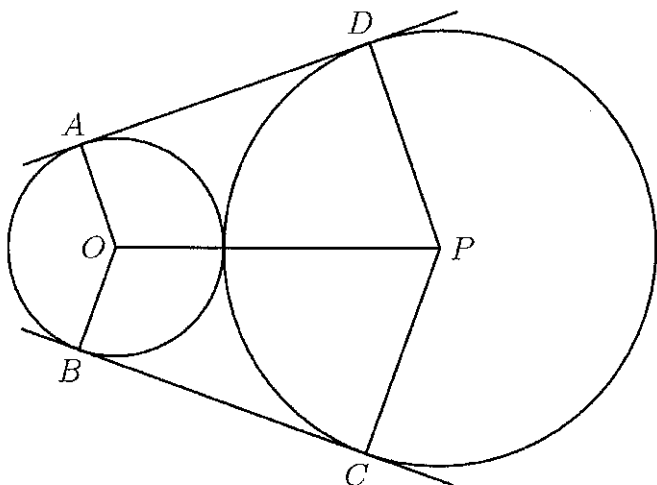


- (A) 15 (B) 17 (C) $\frac{35}{2}$ (D) 18 (E) $\frac{55}{3}$

Problem 24

Circles with centers O and P have radii 2 and 4, respectively, and are externally tangent. Points A and B on the circle with center O and points C and D on the circle with center P are such that AD and BC are common external tangents to the circles. What is the area of the concave hexagon $AOBCPD$?

圆心为 O 和 P 的圆半径分别是 2 和 4，且相互外切。点 A 和 B 在圆 O 上，点 C 和 D 在圆 P 上，满足 AD 和 BC 均是两圆的公切线。那么凸六边形 $AOBCPD$ 的面积是多少？



- (A) $18\sqrt{3}$ (B) $24\sqrt{2}$ (C) 36 (D) $24\sqrt{3}$ (E) $32\sqrt{2}$

Problem 25

Mr. Jones has eight children of different ages. On a family trip his oldest child, who is 9, spots a license plate with a 4-digit number in which each of two digits appears two times. "Look, daddy!!!!" she exclaims. "That number is evenly divisible by the age of each of us kids!" "That's right," replies Mr. Jones, "and the last two digits just happen to be my age." Which of the following is *not* the age of one of Mr. Jones's children?

琼斯先生有八个不同年龄的孩子。在一次家庭旅行中，他最大的孩子，9岁，看到了一个4位数字的车牌，其中两个数字各出现两次。“看，爸爸！！！！”她喊道。“这个数字可以被我们每个孩子的年龄整除！”“没错，”琼斯先生回答说，“最后两位数正好是我的年龄。”以下哪项不是琼斯先生的孩子的年龄？

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8