2007 AMC10A 试题

Problem 1

One ticket to a show costs $\$20$ at full price. Susan buys 4 tickets using a coupon that gives her a 25% discount. Pam buys 5 tickets using a coupon that gives her a 30% discount. How many more dollars does Pam pay than Susan?

某场演出的全价票一张是 20 美元。Susan 用优惠券买了 4 张票，优惠券能给她 25%的折扣。Pam 也使用了优惠券买了 5 张票，她使用的优惠券有 30%的折扣。那么 Pam 比 Susan 多付了多少美元?

(A) 2 (B) 5 (C) 10 (D) 15 (E) 20

Problem 2

Define $a@b = ab - b^2$ and $a\#b = a + b - ab^2$. What is $\dfrac{6@2}{6\#2}$?

定义$a@b = ab - b^2$，且$a\#b = a + b - ab^2$，那么$\dfrac{6@2}{6\#2}$是多少?

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{8}$ (D) $\dfrac{1}{4}$ (E) $\dfrac{1}{2}$

Problem 3

An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base that measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. By how many centimeters does the water rise?

一个水缸的长方形底面尺寸是 100 cm × 40 cm，高为 50 cm. 里面水高为 40 cm. 一块砖底面尺寸是 40 cm × 20 cm, 高 10 cm, 被放入水缸中。问水会升高多少厘米?

(A) 0.5 (B) 1 (C) 1.5 (D) 2 (E) 2.5

Problem 4

The larger of two consecutive odd integers is three times the smaller. What is their sum?

两个连续奇整数中较大的数是较小的数的 3 倍。那么它们的和是多少?

(A) 4 (B) 8 (C) 12 (D) 16 (E) 20

Problem 5

A school store sells 7 pencils and 8 notebooks for $\$4.15$. It also sells 5 pencils and 3 notebooks for $\$1.77$. How much do 16 pencils and 10 notebooks cost?

一家学校商店卖 7 支铅笔和 8 本笔记本总价为 4.15 美元。同时卖 5 支铅笔和 3 本笔记本总价是 1.77 美元。那么 16 支铅笔和 10 本笔记本总价是多少美元?

(A) \$1.76 (B) \$5.84 (C) \$6.00 (D) \$6.16 (E) \$6.32

Problem 6

At Euclid High School, the number of students taking the AMC 10 was 60 in 2002, 66 in 2003, 70 in 2004, 76 in 2005, 78 in 2006, and is 85 in 2007. Between what two consecutive years was there the largest percentage increase?

在欧几里得高中，参加 AMC10 的学生人数在 2002 年是 60 人，在 2003 年是 66 人，在 2004 年是 70 人，在 2005 年是 76 人，在 2006 年是 78 人，在 2007 年是 85 人。在哪两个连续的年份间增加的百分比最大?

(A) 2002 and 2003 (B) 2003 and 2004 (C) 2004 and 2005 (D) 2005 and 2006 (E) 2006 and 2007

Problem 7

Last year Mr. Jon Q. Public received an inheritance. He paid 20% in federal taxes on the inheritance, and paid 10% of what he had left in state taxes. He paid a total of 10500 for both taxes. How many dollars was his inheritance?

去年，Jon Q.Public 先生获得了一笔遗产。他为遗产缴纳了 20%的联邦税，还缴纳了剩余部分的 10%作为州税。两项税总共付了 10500 元。那么他的遗产是多少美元?

(A) 30000 (B) 32500 (C) 35000 (D) 37500 (E) 40000

Problem 8

Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D is inside triangle ABC, angle ABC measures 40 degrees, and angle ADC measures 140 degrees. What is the degree measure of angle BAD?

三角形 ABC 和 ADC 都是等腰三角形，其中$AB = BC$，$AD = DC$。点 D 位于三角形 ABC 内，角 ABC 等于 40 度，且角 ADC 等于 140 度。那么角 BAD 等于多少度?

(A) 20 (B) 30 (C) 40 (D) 50 (E) 60

Problem 9

Real numbers a and b satisfy the equations $3^a = 81^{b+2}$ and $125^b = 5^{a-3}$. What is ab?

实数 a 和 b 满足方程$3^a = 81^{b+2}$以及$125^b = 5^{a-3}$.那么ab是多少?

(A) -60 (B) -17 (C) 9 (D) 12 (E) 60

Problem 10

The Dunbar family consists of a mother, a father, and some children. The average age of the members of the family is 20, the father is 48 years old, and the average age of the mother and children is 16. How many children are in the family?

Dunbar 家族由一位母亲，一位父亲和一些孩子组成。家族所有成员的平均年龄是 20 岁，父亲的年龄是 48 岁，母亲和孩子们的年龄的平均值是 16 岁。则这个家族里共有多少个孩子?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 11

The numbers from 1 to 8 are placed at the vertices of a cube in such a manner that the sum of the four numbers on each face is the same. What is this common sum?

数字 1 到 8 被放到一个正方体的 8 个顶点上，满足每个面上的 4 个数字之和都相同。问这个共同的和是多少?

(A) 14 (B) 16 (C) 18 (D) 20 (E) 24

Problem 12

Two tour guides are leading six tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

两名导游带领着六名游客。导游们决定分开。每位游客必须选择一名导游，但规定每位导游必须至少接待一名游客。有多少种可能的对导游和游客进行分组的方法?

(A) 56 (B) 58 (C) 60 (D) 62 (E) 64

Problem 13

Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium?

Yan 在他家和体育场之间的某个地方。要到达体育场，他可以直接步行到体育场，或者他可以步行回家，然后骑自行车去体育场。已知他骑车的速度是走路的 7 倍，并且两种选择都需要相同的时间。Yan 到他家的距离和到体育场的距离之比是多少?

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{5}{6}$ (E) $\frac{7}{8}$

Problem 14

A triangle with side lengths in the ratio $3:4:5$ is inscribed in a circle with radius 3. What is the area of the triangle?

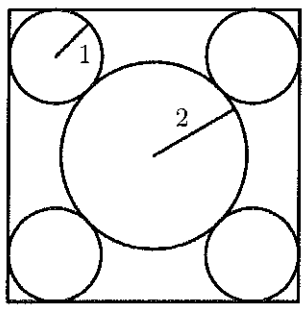
一个三边长的比为$3:4:5$的三角形内接在一个半径为 3 的圆中，那么三角形的面积是多少?

(A) 8.64 (B) 12 (C) 5π (D) 17.28 (E) 18

Problem 15

Four circles of radius 1 are each tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. What is the area of the square?

四个半径为 1 的圆均和一个正方形的两条边相切，且和一个半径为 2 的圆外切，如图所示。那么正方形的面积是多少?



(A) 32 (B) $22+12\sqrt{2}$ (C) $16+16\sqrt{3}$ (D) 48 (E) $36+16\sqrt{2}$

Problem 16

Integers a, b, c, and d, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that $ad-bc$ is even?

从 0 到 2007(包括 0 和 2007)范围内独立且随机的挑选 4 个不必不同的整数$a, b, c,$和d. 问$ad-bc$是偶数的概率是多少?

(A) $\frac{3}{8}$ (B) $\frac{7}{16}$ (C) $\frac{1}{2}$ (D) $\frac{9}{16}$ (E) $\frac{5}{8}$

Problem 17

Suppose that m and n are positive integers such that $75m=n^3$. What is the minimum possible value of $m+n$?

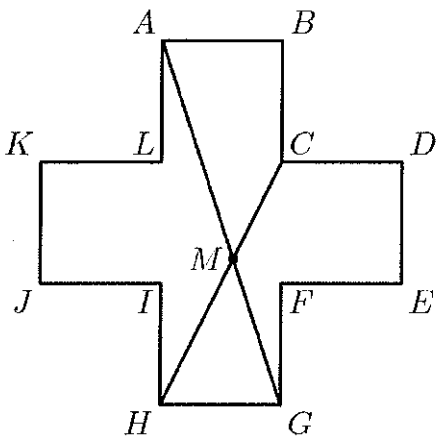
假设m 和 n均是正整数，满足$75m=n^3$。则$m+n$的最小可能值是多少?

(A) 15 (B) 30 (C) 50 (D) 60 (E) 5700

Problem 18

Consider the 12-sided polygon $ABCDEFGHIJKL$, as shown. Each of its sides has length 4, and each two consecutive sides form a right angle. Suppose that $\overline{AG}$ and $\overline{CH}$ meet at M. What is the area of quadrilateral $ABCM$?

考虑一个如图所示的 12 边形$ABCDEFGHIJKL$。每条边长度均为 4，并且每两条连续的边形成一个直角。假设$\overline{AG}$交$\overline{CH}$于 M。则四边形$ABCM$的面积是多少？

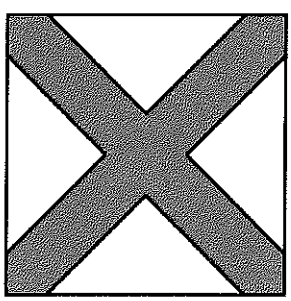


(A) $\frac{44}{3}$ (B) 16 (C) $\frac{88}{5}$ (D) 20 (E) $\frac{62}{3}$

Problem 19

A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. What is the ratio of the side length of the square to the brush width?

将画笔扫过正方形的两条对角线，就生成了如图所示对称的绘制区域。已知正方形面积的一半被画笔绘制。那么正方形的边长与画笔宽度的比值是多少？



(A) $2\sqrt{2}+1$ (B) $3\sqrt{2}$ (C) $2\sqrt{2}+2$ (D) $3\sqrt{2}+1$ (E) $3\sqrt{2}+2$

Problem 20

Suppose that the number a satisfies the equation $4 = a + a^{-1}$. What is the value of $a^4 + a^{-4}$?

假设数a满足方程$4 = a + a^{-1}$，则$a^4 + a^{-4}$的值是多少?

(A) 164　　(B) 172　　(C) 192　　(D) 194　　(E) 212

Problem 21

A sphere is inscribed in a cube that has a surface area of 24 square meters. A second cube is then inscribed within the sphere. What is the surface area in square meters of the inner cube?

一个球位于一个表面积为 24 平方米的正方体内且和正方体内切。球内有个正方体且球和此正方体外接。则内部正方体的表面积是多少平方米?

(A) 3　　(B) 6　　(C) 8　　(D) 9　　(E) 12

Problem 22

A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with the terms 247, 475, and 756 and end with the term 824. Let S be the sum of all the terms in the sequence. What is the largest prime factor that always divides S?

一个各项均为三位整数的有限数列有这样的性质：每项的十位和个位数字都分别等于下一项的百位和十位数字，并且最后一项的十位和个位数字分别等于第一项的百位和十位数字。例如，这样一个数列前几项可能是 247, 475, 和 756，以 824 结尾。令 S 表示这个数列的所有项之和。那么总是能整除 S 的最大质因数是多少?

(A) 3　　(B) 7　　(C) 13　　(D) 37　　(E) 43

Problem 23

How many ordered pairs (m, n) of positive integers, with $m \geq n$, have the property that their squares differ by 96?

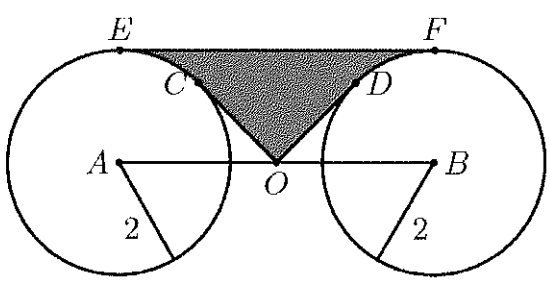
有多少对有序正整数对(m,n)，其中$m \geq n$，满足这样的性质：它们的平方差为 96.

(A) 3　(B) 4　(C) 6　(D) 9　(E) 12

Problem 24

Circles centered at A and B each have radius 2, as shown. Point O is the midpoint of $\overline{AB}$, and $OA = 2\sqrt{2}$. Segments OC and OD are tangent to the circles centered at A and B, respectively, and EF is a common tangent. What is the area of the shaded region $ECODF$?

圆心在点 A 和点 B 的两个圆半径均为 2，如图所示。点 O 是$\overline{AB}$的中点，且$OA = 2\sqrt{2}$。线段 OC 和 OD 分别和圆 A、圆 B 相切，EF 为两圆公切线。则阴影部分区域$ECODF$的面积是多少?



(A) $\frac{8\sqrt{2}}{3}$　(B) $8\sqrt{2}-4-\pi$　(C) $4\sqrt{2}$　(D) $4\sqrt{2}+\frac{\pi}{8}$　(E) $8\sqrt{2}-2-\frac{\pi}{2}$

Problem 25

For each positive integer n, let $S(n)$ denote the sum of the digits of n. For how many values of n is $n + S(n) + S(S(n)) = 2007$?

对于每个正整数n，令$S(n)$表示n的各个位数字之和。则n有多少个值使得$n + S(n) + S(S(n)) = 2007$?

(A) 1　(B) 2　(C) 3　(D) 4　(E) 5