

2007 AMC10B 试题

Problem 1

Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?

Isabella 的房间有 3 个卧室。每个卧室长为 12 英尺, 宽 10 英尺, 高 8 英尺。Isabella 现在要把这所有卧室的墙壁都粉刷一遍。每间卧室的门道和窗户不会粉刷, 而这些地方在每个房间的面积均为 60 平方英尺。那么要粉刷的墙壁总共是多少平方英尺?

- (A) 678 (B) 768 (C) 786 (D) 867 (E) 876

Problem 2

Define the operation $\star$ by $a \star b = (a + b)b$. What is $(3 \star 5) - (5 \star 3)$?

定义操作符 $\star$ 如下: $a \star b = (a + b)b$. 则表达式 $(3 \star 5) - (5 \star 3)$ 的值是多少?

- (A) -16 (B) -8 (C) 0 (D) 8 (E) 16

Problem 3

A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?

一名大学生周末开着他的小型车回家, 行驶 120 英里, 平均每加仑汽油可以开 30 英里。在回程中, 这名学生驾驶父母的 SUV, 平均每加仑汽油仅行驶 20 英里。则往返的平均汽油里程是多少? 以英里每加仑为单位。

- (A) 22 (B) 24 (C) 25 (D) 26 (E) 28

Problem 4

The point O is the center of the circle circumscribed about $\triangle ABC$, with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ$. What is the degree measure of $\angle ABC$?

点 O 是 $\triangle ABC$ 外接圆的圆心, 其中 $\angle BOC = 120^\circ$, $\angle AOB = 140^\circ$ 。那么

$\angle ABC$ 是多少度?

- (A) 35 (B) 40 (C) 45 (D) 50 (E) 60

Problem 5

In a certain land, all Arogs are Brafs, all Crups are Brafs, all Dramps are Arogs, and all Crups are Dramps. Which of the following statements is implied by these facts?

在某一片土地上, 所有的 Arogs 都是 Brafs, 所有的 Dramps 都是 Arogs, 所有的 Crups 都是 Dramps. 下面哪个论断可由这些事实得到?

- (A) 所有的 Dramps 都是 Brafs 和 Crups。
(B) 所有的 Brafs 都是 Crups 和 Dramps。
(C) 所有 Arogs 都是 Crups 和 Dramps。
(D) 所有的 Crups 都是 Arogs 和 Brafs。
(E) 所有的 Arogs 都是 Dramps, 并且一些 Arogs 可能不是 Crups。

Problem 6

The 2007 AMC 10 will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave only the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

2007 年 AMC10 的判分规则如下: 答对一题得 6 分, 答错得 0 分, 不答得 1.5 分。

Sarah 在浏览完 25 题后, 决定尝试前 22 题, 最后 3 题不答。为了得到至少 100 分, 那么前 22 题她必须答对多少题?

- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

Problem 7

All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90^\circ$. What is the degree measure of $\angle E$?

凸五边形 $ABCDE$ 的所有边等长, 且 $\angle A = \angle B = 90^\circ$. 那么 $\angle E$ 的度数是多少度?

- (A) 90 (B) 108 (C) 120 (D) 144 (E) 150

Problem 8

On the trip home from the meeting where this AMC10 was constructed, the Contest Chair noted that his airport parking receipt had digits of the form $bbcac$, where $0 \leq a < b < c \leq 9$, and b was the average of a and c . How many different five-digit numbers satisfy all these properties?

这次 AMC10 的出题会议结束后回家的路上, 竞赛主席注意到, 他的机场停车收据上有 $bbcac$ 这种形式的数字, 其中 $0 \leq a < b < c \leq 9$, 且 b 是 a 和 c 的平均值。

有多少个不同的五位数满足所有这些属性?

- (A) 12 (B) 16 (C) 18 (D) 20 (E) 25

Problem 9

A cryptographic code is designed as follows. The first time a letter appears in a given message it is replaced by the letter that is 1 place to its right in the alphabet (assuming that the letter A is one place to the right of the letter Z). The second time this same letter appears in the given message, it is replaced by the letter that is $1 + 2$ places to the right, the third time it is replaced by the letter that is $1 + 2 + 3$ places to the right, and so on. For example, with this code the word "banana" becomes "cbodqg". What letter will replace the last letter s in the message

"Lee's sis is a Mississippi miss, Chriss!"?

一种密码的设计如下。当一个字母第一次出现在给定的消息中时, 它会被字母表中位于其右侧的第一个字母所取代(假设字母 A 位于字母 Z 右侧的第一个位置)。同一个字母第二次出现在给定的消息中时, 会被这个字母右边第 $1 + 2$ 个字母替换, 第三次出现会被这个字母右边第 $1 + 2 + 3$ 个字母替换, 依此类推。例如,

在这种加密方式下, “banana”一词变成了“cbodqg”。问下面消息中的最后一个字母 s 会被哪个字母代替?

”Lee’s sis is a Mississippi miss, Chriss!”?

- (A) g (B) h (C) o (D) s (E) t

Problem 10

Two points B and C are in a plane. Let S be the set of all points A in the plane for which $\triangle ABC$ has area 1. Which of the following describes S ?

- (A) two parallel lines (B) a parabola (C) a circle (D) a line segment (E) two points

点 B 和 C 在同一平面内。令 S 为使得 $\triangle ABC$ 面积为 1 的所有这样的点 A 的集合。下面哪个描述了 S ?

- (A) 两条平行线 (B) 一个抛物线 (C) 一个圆 (D) 一条线段 (E) 两个点

Problem 11

A circle passes through the three vertices of an isosceles triangle that has two sides of length 3 and a base of length 2. What is the area of this circle?

一个等腰三角形两条腰长均为 3, 底边长为 2, 一个圆通过这个三角形的三个顶点。则这个圆的面积是多少?

- (A) 2π (B) $\frac{5}{2}\pi$ (C) $\frac{81}{32}\pi$ (D) 3π (E) $\frac{7}{2}\pi$

Problem 12

Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N ?

Tom 的年龄是 T 岁, 他的年龄是他的三个孩子的年龄之和。 N 年前他的年龄是他们年龄之和的 2 倍。则 T/N 是多少?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 13

Two circles of radius 2 are centered at $(2, 0)$ and at $(0, 2)$. What is the area of the intersection of the interiors of the two circles?

两个半径为 2 的圆的圆心分别在 $(2, 0)$ 和 $(0, 2)$, 那么这两个圆内部相交的区域的面积是多少?

- (A) $\pi - 2$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi\sqrt{3}}{3}$ (D) $2(\pi - 2)$ (E) π

Problem 14

Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?

一些男孩和女孩正在洗车, 为班级去中国旅游筹集资金。最初, 这个群体 40% 的成员是女孩。不久之后, 两个女孩离开, 两个男孩加入, 之后这群人中 30% 是女孩。问这个群体内一开始有多少个女孩?

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12

Problem 15

The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?

四边形 $ABCD$ 中, $\angle A = 2\angle B = 3\angle C = 4\angle D$. 那么 $\angle A$ 的度数是多少? 结果精确到整数.

- (A) 125 (B) 144 (C) 153 (D) 173 (E) 180

Problem 16

A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

一个班上有 10% 是 11 年级学生, 90% 是 12 年级学生。一位老师给这个班做了次测试。测试的平均分是 84 分。11 年级学生全部得到了相同的分数, 12 年级学生的平均分是 83 分。则这次测试每个 11 年级学生得到了多少分?

- (A) 85 (B) 88 (C) 93 (D) 94 (E) 98

Problem 17

Point P is inside equilateral $\triangle ABC$. Points Q , R , and S are the feet of the perpendiculars from P to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Given that $PQ = 1$, $PR = 2$, and $PS = 3$, what is AB ?

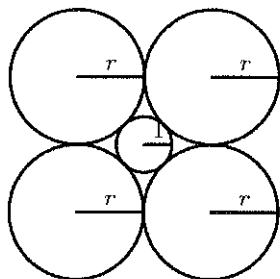
点 P 是等边 $\triangle ABC$ 内一点。点 Q , R , 和 S 分别是从小 P 向边 $\overline{AB}$, $\overline{BC}$, 和 $\overline{CA}$ 所引垂线的垂足。已知 $PQ = 1$, $PR = 2$, $PS = 3$, 那么 AB 是多长?

- (A) 4 (B) $3\sqrt{3}$ (C) 6 (D) $4\sqrt{3}$ (E) 9

Problem 18

A circle of radius 1 is surrounded by 4 circles of radius r as shown. What is r ?

如图所示, 一个半径为 1 的圆被 4 个半径为 r 的圆所包围。则 r 是多少?

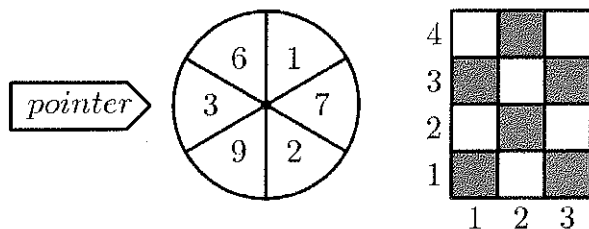


- (A) $\sqrt{2}$ (B) $1 + \sqrt{2}$ (C) $\sqrt{6}$ (D) 3 (E) $2 + \sqrt{2}$

Problem 19

The wheel shown is spun twice, and the randomly determined numbers opposite the pointer are recorded. The first number is divided by 4, and the second number is divided by 5. The first remainder designates a column, and the second remainder designates a row on the checkerboard shown. What is the probability that the pair of numbers designates a shaded square?

如图所示的轮子被转了 2 次，并且随机确定的和指针相对的数字被记录了下来。
 将第 1 个数除以 4，第 2 个数除以 5。在如图所示的棋盘上，第 1 个余数指定一
 列，第 2 个余数指定一行。则这对余数所确定的方格是个阴影方格的概率是多少？



- (A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{5}{9}$ (E) $\frac{2}{3}$

Problem 20

A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

将 25 个方块放入 5×5 的方格中。从方格中选择 3 个方块，满足这 3 个方块中不存在某两个是在同一行或同一列，一共有多少种方法？

- (A) 100 (B) 125 (C) 600 (D) 2300 (E) 3600

Problem 21

Right $\triangle ABC$ has $AB = 3$, $BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on $\overline{AC}$, W on $\overline{AB}$, and Z on $\overline{BC}$. What is the side length of the square?

直角三角形 $\triangle ABC$ 中， $AB = 3$, $BC = 4$, $AC = 5$. 正方形 $XYZW$ 内接在 $\triangle ABC$ 中，满足 X 和 Y 在 AC 上， W 在 AB 上， Z 在 BC 上。则这个正方形的边长是多少？

- (A) $\frac{3}{2}$ (B) $\frac{60}{37}$ (C) $\frac{12}{7}$ (D) $\frac{23}{13}$ (E) 2

Problem 22

A player chooses one of the numbers 1 through 4. After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through 4. If the number chosen appears on the bottom of exactly one die after it has been rolled, then the player wins 1 dollar. If the number chosen appears on the bottom of both of the dice, then the player wins 2 dollars. If the number chosen does not appear on the bottom of either of the dice, the player loses 1 dollar. What is the expected return to the player, in dollars, for one roll of the dice?

一个选手从 1 到 4 中选择一个数字。选完后，接着掷两枚 4 面（四面体）的骰子，骰子的四个面标有 1 到 4。如果选手一开始选择的数字出现在恰好其中一个骰子的底面，那么就赢得 1 美元。如果选择的数字在 2 个骰子的底面都出现，那么就赢得 2 美元。如果选择的数字没出现在任何一个骰子的底面，那么就输掉 1 美元。

问选手将 2 枚骰子只扔一次，所得回报的期望值是多少美元？

- (A) $-\frac{1}{8}$ (B) $-\frac{1}{16}$ (C) 0 (D) $\frac{1}{16}$ (E) $\frac{1}{8}$

Problem 23

A pyramid with a square base is cut by a plane that is parallel to its base and 2 units from the base. The surface area of the smaller pyramid that is cut from the top is half the surface area of the original pyramid. What is the altitude of the original pyramid?

一个底面为正方形的四棱锥被平行于底面且距离底面 2 个单位的平面所截。从顶部截下来的较小的四棱锥的表面积是原来四棱锥的表面积的一半。则原来四棱锥的高是多少？

- (A) 2 (B) $2 + \sqrt{2}$ (C) $1 + 2\sqrt{2}$ (D) 4 (E) $4 + 2\sqrt{2}$

Problem 24

Let n denote the smallest positive integer that is divisible by both 4 and 9 , and whose base-10 representation consists of only 4's and 9's, with at least one of each. What are the last four digits of n ?

令 n 是满足以下条件的最小正整数: 能同时被 4 和 9 整除, 且十进制表示只由数字 4 和 9 组成, 其中 4 和 9 均至少出现 1 次。则 n 的最后 4 位数字是多少?

- (A) 4444 (B) 4494 (C) 4944 (D) 9444 (E) 9944

Problem 25

How many pairs of positive integers (a, b) are there such that a and b have no common factors greater than 1 and $\frac{a}{b} + \frac{14b}{9a}$ is an integer?

有多少对正整数 (a, b) , 满足 a 和 b 没有大于 1 的公因子, 且 $\frac{a}{b} + \frac{14b}{9a}$ 是个整数?

- (A) 4 (B) 6 (C) 9 (D) 12 (E) 无穷多个