

2008 AMC10A 试题

Problem 1

A bakery owner turns on his doughnut machine at 8:30 AM. At 11:10 AM the machine has completed one third of the day's job. At what time will the doughnut machine complete the job?

- (A) 1:50 PM (B) 3:00 PM (C) 3:30 PM (D) 4:30 PM (E) 5:50 PM

一位面包店老板在早上 8:30 打开他的甜甜圈机器。到早上 11:10, 这台机器已经完成了一天工作量的三分之一。问甜甜圈机什么时候能完成一天的工作?

- (A) 下午 1:50 (B) 下午 3:00 (C) 下午 3:30 (D) 下午 4:30 (E) 下午 5:50

Problem 2

A square is drawn inside a rectangle. The ratio of the width of the rectangle to a side of the square is 2 : 1. The ratio of the rectangle's length to its width is 2 : 1. What percent of the rectangle's area is in the square?

一个长方形内部画了一个正方形。长方形的宽和正方形的边长之比为 2 : 1。长方形的长和宽之比为 2 : 1。问长方形面积的百分之多少在正方形内?

- (A) 12.5 (B) 25 (C) 50 (D) 75 (E) 87.5

Problem 3

For the positive integer n , let $\langle n \rangle$ denote the sum of all the positive divisors of n with the exception of n itself. For example,

$$\langle 4 \rangle = 1 + 2 = 3 \text{ and } \langle 12 \rangle = 1 + 2 + 3 + 4 + 6 = 16. \text{ What is } \langle \langle \langle 6 \rangle \rangle \rangle?$$

对于一个正整数 n , 定义 $\langle n \rangle$ 为 n 的所有除了 n 本身之外的正因子之和。例如,

$$\langle 4 \rangle = 1 + 2 = 3, \quad \langle 12 \rangle = 1 + 2 + 3 + 4 + 6 = 16. \text{ 那么 } \langle \langle \langle 6 \rangle \rangle \rangle \text{ 是多少?}$$

- (A) 6 (B) 12 (C) 24 (D) 32 (E) 36

Problem 4

Suppose that $\frac{2}{3}$ of 10 bananas are worth as much as 8 oranges. How many oranges are worth as much as $\frac{1}{2}$ of 5 bananas?

假设 10 根香蕉总价格的 $\frac{2}{3}$ 等于 8 只橘子的总价。那么多少只橘子的总价等于 5 根香蕉总价的 $\frac{1}{2}$?

- (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) 4

Problem 5

Which of the following is equal to the product

$$\frac{8}{4} \cdot \frac{12}{8} \cdot \frac{16}{12} \cdots \frac{4n+4}{4n} \cdots \frac{2008}{2004}?$$

下面哪个等于乘积

$$\frac{8}{4} \cdot \frac{12}{8} \cdot \frac{16}{12} \cdots \frac{4n+4}{4n} \cdots \frac{2008}{2004}?$$

- (A) 251 (B) 502 (C) 1004 (D) 2008 (E) 4016

Problem 6

A triathlete competes in a triathlon in which the swimming, biking, and running segments are all of the same length. The triathlete swims at a rate of 3 kilometers per hour, bikes at a rate of 20 kilometers per hour, and runs at a rate of 10 kilometers per hour. Which of the following is closest to the triathlete's average speed, in kilometers per hour, for the entire race?

一名铁人三项运动员参加一项铁人三项比赛，其中游泳、骑自行车和跑步的路程长度相同。铁人三项运动员以每小时 3 公里的速度游泳，以每小时 20 公里的速度骑自行车，以每小时 10 公里的速度跑步。以下哪项最接近铁人三项运动员在整个比赛中的平均速度（以千米/小时为单位）？

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Problem 7

The fraction $\frac{(3^{2008})^2 - (3^{2006})^2}{(3^{2007})^2 - (3^{2005})^2}$ simplifies to which of the following?

分数 $\frac{(3^{2008})^2 - (3^{2006})^2}{(3^{2007})^2 - (3^{2005})^2}$ 的化简结果是下面哪个?

- (A) 1 (B) $\frac{9}{4}$ (C) 3 (D) $\frac{9}{2}$ (E) 9

Problem 8

Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a \$90 rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves \$15 by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?

Heather 比较了 2 个不同商店的同一款新电脑的价格。商店 A 在标价的基础上降低 15%，并且提供 90 美元回扣。商店 B 在同样标价的基础上降低 25% 但没有回扣。Heather 在商店 A 买了电脑，比在商店 B 买节省了 15 美元。问电脑的标价是多少美元？

- (A) 750 (B) 900 (C) 1000 (D) 1050 (E) 1500

Problem 9

Suppose that $\frac{2x}{3} - \frac{x}{6}$ is an integer. Which of the following statements must be true about x ?

- (A) It is negative.
 (B) It is even, but not necessarily a multiple of 3.
 (C) It is a multiple of 3, but not necessarily even.
 (D) It is a multiple of 6, but not necessarily a multiple of 12.
 (E) It is a multiple of 12.

假设 $\frac{2x}{3} - \frac{x}{6}$ 是个整数。那么关于 x 的说法下面哪个是正确的？

- (A) 它是个负数

- (B) 它是个偶数，但不一定是 3 的倍数
- (C) 它是 3 的倍数，但不一定是偶数
- (D) 它是 6 的倍数，但不一定是 12 的倍数。
- (E) 它是 12 的倍数。

Problem 10

Each of the sides of a square S_1 with area 16 is bisected, and a smaller square S_2 is constructed using the bisection points as vertices. The same process is carried out on S_2 to construct an even smaller square S_3 . What is the area of S_3 ?

一个面积为 16 的正方形 S_1 的每条边都被平分，以这些平分点作为顶点，作出一个小的正方形 S_2 。在 S_2 上再进行同样的操作作出一个更小的正方形 S_3 。问 S_3 的面积是多少？

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 3 (E) 4

Problem 11

While Steve and LeRoy are fishing 1 mile from shore, their boat springs a leak, and water comes in at a constant rate of 10 gallons per minute. The boat will sink if it takes in more than 30 gallons of water. Steve starts rowing toward the shore at a constant rate of 4 miles per hour while LeRoy bails water out of the boat. What is the slowest rate, in gallons per minute, at which LeRoy can bail if they are to reach the shore without sinking?

当 Steve 和 LeRoy 在离海岸 1 英里的地方钓鱼时，他们的船突然漏水，水以每分钟 10 加仑的恒定速度流入。如果船进水超过 30 加仑，就会沉没。Steve 开始以每小时 4 英里的恒定速度向岸边划船，而 LeRoy 则同时将水排到船外。如果要保证船在到达海岸前不沉没，则 LeRoy 排水的最低速度是多少加仑每分钟？

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

Problem 12

In a collection of red, blue, and green marbles, there are 25% more red marbles than blue marbles, and there are 60% more green marbles than red marbles. Suppose that there are r red marbles. What is the total number of marbles in the collection?

在一堆红色，蓝色和绿色的玻璃球中，红球比蓝球多25%，绿球比红球多60%。

假设有 r 个红球。那么这堆球的总个数是多少？

- (A) $2.85r$ (B) $3r$ (C) $3.4r$ (D) $3.85r$ (E) $4.25r$

Problem 13

Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t ?

Doug 将一间房间粉刷完需要 5 小时。Dave 将同样的房间粉刷完需要 7 小时。现

在 Doug 和 Dave 一起粉刷这间房间，期间休息 1 小时用于吃午饭。令 t 表示他们

合作完成这项任务所花的总小时数，包括午饭时间。那么下面哪个是满足 t 的方

程？

- (A) $\left(\frac{1}{5} + \frac{1}{7}\right)(t+1) = 1$ (B) $\left(\frac{1}{5} + \frac{1}{7}\right)t + 1 = 1$ (C) $\left(\frac{1}{5} + \frac{1}{7}\right)t = 1$
(D) $\left(\frac{1}{5} + \frac{1}{7}\right)(t-1) = 1$ (E) $(5+7)t = 1$

Problem 14

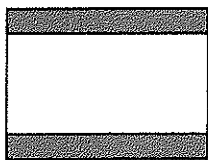
Older television screens have an aspect ratio of 4 : 3. That is, the ratio of the width to the height is 4 : 3. The aspect ratio of many movies is not 4 : 3, so they are sometimes shown on a television screen by "letterboxing" - darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of 2 : 1 and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?

老式电视屏幕的纵横比为4 : 3。也就是说，宽度与高度之比为4 : 3。然而许多电

影的纵横比都不是4 : 3，所以它们有时会在电视屏幕上通过“字母盒”

(letterboxing) 来显示, 如图所示, 在屏幕的顶部和底部都有相同高度的暗条。

假设一部电影的纵横比为 $2:1$, 并且在一个对角线长度为 27 英寸的老式电视屏幕上播放。则每个暗条的高度是多少英寸?



- (A) 2 (B) 2.25 (C) 2.5 (D) 2.7 (E) 3

Problem 15

Yesterday Han drove 1 hour longer than Ian at an average speed 5 miles per hour faster than Ian. Jan drove 2 hours longer than Ian at an average speed 10 miles per hour faster than Ian. Han drove 70 miles more than Ian. How many more miles did Jan drive than Ian?

昨天 Han 的开车速度比 Ian 快 5 英里每小时, 且开车时间比 Ian 长 1 小时。Jan 的开车速度比 Ian 快 10 英里每小时, 且开车时间比 Ian 长 2 小时。已知 Han 比 Ian 多开了 70 英里。那么 Jan 比 Ian 多开多少英里?

- (A) 120 (B) 130 (C) 140 (D) 150 (E) 160

Problem 16

Points A and B lie on a circle centered at O , and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

点 A 和 B 在圆心为 O 的圆上, 满足 $\angle AOB = 60^\circ$ 。第二个圆和第一个圆内切并且和线段 $\overline{OA}$ 、 $\overline{OB}$ 都相切。则小圆的面积和大圆的面积的比值是多少?

- (A) $\frac{1}{16}$ (B) $\frac{1}{9}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$

Problem 17

An equilateral triangle has side length 6. What is the area of the region containing all points that are outside the triangle but not more than 3 units from a point on the triangle?

一个等边三角形的边长为 6. 则位于三角形外, 且距离三角形上的点不超过 3 个单位长的所有点所组成的区域的面积是多少?

- (A) $36 + 24\sqrt{3}$ (B) $54 + 9\pi$ (C) $54 + 18\sqrt{3} + 6\pi$ (D) $(2\sqrt{3} + 3)^2 \pi$
 (E) $9(\sqrt{3} + 1)^2 \pi$

Problem 18

A right triangle has perimeter 32 and area 20. What is the length of its hypotenuse?

一个直角三角形的周长为 32, 面积是 20. 则它的斜边长是多少?

- (A) $\frac{57}{4}$ (B) $\frac{59}{4}$ (C) $\frac{61}{4}$ (D) $\frac{63}{4}$ (E) $\frac{65}{4}$

Problem 19

Rectangle $PQRS$ lies in a plane with $PQ = RS = 2$ and $QR = SP = 6$. The rectangle is rotated 90° clockwise about R , then rotated 90° clockwise about the point S moved to after the first rotation. What is the length of the path traveled by point P ?

矩形 $PQRS$ 在某个平面内, 且 $PQ = RS = 2$, $QR = SP = 6$. 先将矩形

绕点 R 顺时针旋转 90° , 然后再绕着第一次旋转后点 S 移动到的点顺时针旋转 90° .

则点 P 所走过的路径的长度是多少?

- (A) $(2\sqrt{3} + \sqrt{5})\pi$ (B) 6π (C) $(3 + \sqrt{10})\pi$ (D) $(\sqrt{3} + 2\sqrt{5})\pi$
 (E) $2\sqrt{10}\pi$

Problem 20

Trapezoid $ABCD$ has bases $\overline{AB}$ and $\overline{CD}$ and diagonals intersecting at K . Suppose that $AB = 9$, $DC = 12$, and the area of $\triangle AKD$ is 24. What is the area of trapezoid $ABCD$?

梯形 $ABCD$ 的底为 $\overline{AB}$ 和 $\overline{CD}$, 对角线交于 K . 假设 $AB = 9$, $DC = 12$, 且

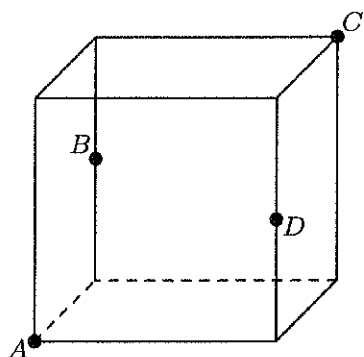
$\triangle AKD$ 的面积为 24. 则梯形 $ABCD$ 的面积是多少?

- (A) 92 (B) 94 (C) 96 (D) 98 (E) 100

Problem 21

A cube with side length 1 is sliced by a plane that passes through two diagonally opposite vertices A and C and the midpoints B and D of two opposite edges not containing A or C , as shown. What is the area of quadrilateral $ABCD$?

一个棱长为 1 的正方体被一个平面所切割, 这个平面通过一条对角线的两个端点 A 和 C , 以及不包含 A 或 C 的两条对棱的中点 B 和 D , 如图所示。则四边形 $ABCD$ 的面积是多少?



- (A) $\frac{\sqrt{6}}{2}$ (B) $\frac{5}{4}$ (C) $\sqrt{2}$ (D) $\frac{5}{8}$ (E) $\frac{3}{4}$

Problem 22

Jacob uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If it comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer?

Jacob 按照以下步骤写出一列数。首先他选择 6 作为首项。之后他通过扔硬币生成后面的每一项。若硬币是正面朝上, 他就将前一项乘 2 再减 1。若反面朝上, 则是取前一项的一半再减 1。问 Jacob 所得到的数列的第 4 项是个整数的概率是多少?

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{3}{4}$

Problem 23

Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly two elements. In how many ways can this be done, assuming that the order in which the subsets are chosen does not matter?

从集合 $S = \{a, b, c, d, e\}$ 中选择两个子集, 满足这两个子集的并集是 S , 且它们的交集包含恰好 2 个元素。问有多少种选择这 2 个子集的方法? 假设并不关心这 2 个子集选出的先后顺序。

- (A) 20 (B) 40 (C) 60 (D) 160 (E) 320

Problem 24

Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

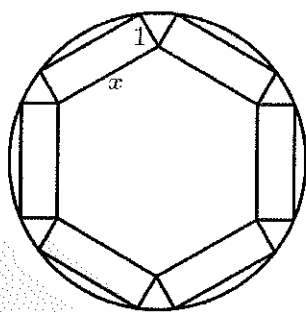
令 $k = 2008^2 + 2^{2008}$. 则 $k^2 + 2^k$ 的个位数字是多少?

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Problem 25

A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x . Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x ?

一张圆桌半径为 4. 桌子上放着六个长方形的垫子。每个位置垫的宽度为 1 和长度为 x , 如图所示。它们是按照这种方式放置的: 每个垫子的两个角和桌子边缘对齐, 这两个角是长度为 x 的边的同一侧的两个端点。此外, 垫子靠内的两个角各自和相邻的垫子的一个靠内的角相接触。则 x 是多少?



(A) $2\sqrt{5} - \sqrt{3}$ (B) 3 (C) $\frac{3\sqrt{7} - \sqrt{3}}{2}$ (D) $2\sqrt{3}$ (E) $\frac{5 + 2\sqrt{3}}{2}$