

2009 AMC10A 试题

Problem 1

One can holds 12 ounces of soda. What is the minimum number of cans needed to provide a gallon (128 ounces) of soda?

一个罐子可以装 12 盎司的苏打水。要装 1 加仑 (128 盎司) 的苏打水, 最少需要多少罐子?

- (A) 7 (B) 8 (C) 9 (D) 10 (E) 11

Problem 2

Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes and quarters. Which of the following could *not* be the total value of the four coins, in cents?

从一个装有一分便士、五分镍币、一角硬币和 25 分硬币的小猪存钱罐中挑选出四枚硬币。以下哪一项不可能是四枚硬币的总价值 (以美分计)?

- (A) 15 (B) 25 (C) 35 (D) 45 (E) 55

Problem 3

Which of the following is equal to $1 + \frac{1}{1 + \frac{1}{1+1}}$?

下面哪个等于 $1 + \frac{1}{1 + \frac{1}{1+1}}$?

- (A) $\frac{5}{4}$ (B) $\frac{3}{2}$ (C) $\frac{5}{3}$ (D) 2 (E) 3

Problem 4

Eric plans to compete in a triathlon. He can average 2 miles per hour in the $\frac{1}{4}$ -mile swim and 6 miles per hour in the 3-mile run. His goal is to finish the triathlon in 2 hours. To accomplish his goal what must his average speed in miles per hour, be for the 15-mile bicycle ride?

Eric 计划参加一次三项全能比赛。他能以 2 英里每小时的速度游泳 $\frac{1}{4}$ 英里，以及 6 英里每小时的速度跑 3 英里。他的目标是在 2 小时内完成三项全能。为了达到他的目标，他骑自行车时需要以多少英里每小时的速度骑完 15 英里？

- (A) $\frac{120}{11}$ (B) 11 (C) $\frac{56}{5}$ (D) $\frac{45}{4}$ (E) 12

Problem 5

What is the sum of the digits of the square of 111, 111, 111?

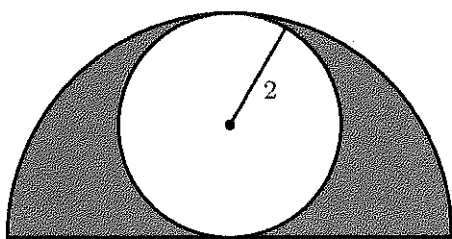
111, 111, 111 的平方的各个位上数字之和是多少？

- (A) 18 (B) 27 (C) 45 (D) 63 (E) 81

Problem 6

A circle of radius 2 is inscribed in a semicircle, as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?

一个半径为 2 的圆如图内接在一个半圆中。阴影部分为半圆内部而在圆外部的区域。问阴影部分所占半圆面积的比例是多少？



- (A) $\frac{1}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{2}{\pi}$ (D) $\frac{2}{3}$ (E) $\frac{3}{\pi}$

Problem 7

A carton contains milk that is 2% fat, an amount that is 40% less fat than the amount contained in a carton of whole milk. What is the percentage of fat in whole milk?

一盒牛奶含有 2% 的脂肪，这个量比一盒全脂牛奶所含脂肪量少 40%。问全脂牛奶所含脂肪的百分比是多少？

- (A) $\frac{12}{5}$ (B) 3 (C) $\frac{10}{3}$ (D) 38 (E) 42

Problem 8

Three generations of the Wen family are going to the movies, two from each generation. The two members of the youngest generation receive a 50% discount as children. The two members of the oldest generation receive a 25% discount as senior citizens. The two members of the middle generation receive no discount. Grandfather Wen, whose senior ticket costs \$6.00, is paying for everyone. How many dollars must he pay?

温家族的三代人将去电影院看电影，每代人 2 个人。最年轻的一代的两个人作为儿童享受 50% 的折扣，最年长的一代的两个人作为老年人享受 25% 的折扣。中间那代的两个人没有任何折扣。温爷爷的老年票需支付 6 美元，而且他将为每个人支付票价。问他总共需要支付多少美元？

- (A) 34 (B) 36 (C) 42 (D) 46 (E) 48

Problem 9

Positive integers a , b , and 2009, with $a < b < 2009$, form a geometric sequence with an integer ratio. What is a ?

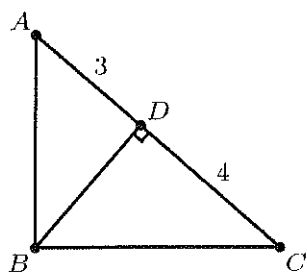
正整数 a , b , 2009 形成一个公比为整数的等比数列，其中 $a < b < 2009$ 。求 a 是多少？

- (A) 7 (B) 41 (C) 49 (D) 289 (E) 2009

Problem 10

Triangle ABC has a right angle at B . Point D is the foot of the altitude from B , $AD = 3$, and $DC = 4$. What is the area of $\triangle ABC$?

三角形 ABC 的直角顶点为 B 。点 D 是从顶点 B 引出的高的垂足。 $AD = 3$, $DC = 4$, 则 $\triangle ABC$ 的面积是多少？



- (A) $4\sqrt{3}$ (B) $7\sqrt{3}$ (C) 21 (D) $14\sqrt{3}$ (E) 421

Problem 11

One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

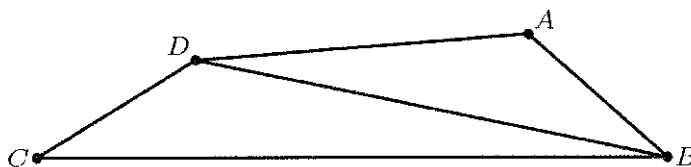
一个正方体的一个维度增加 1, 另一个维度减少 1, 第三个维度不变。则新的长方体的体积比原来正方体的体积小 5。问原来正方体的体积是多少?

- (A) 8 (B) 27 (C) 64 (D) 125 (E) 216

Problem 12

In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD ?

在四边形 $ABCD$ 中, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, 且 BD 是个整数。则 BD 是多少?



- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

Problem 13

Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n) ?

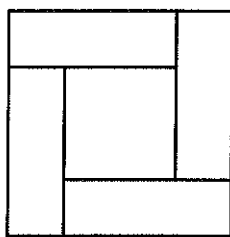
假设 $P = 2^m$, $Q = 3^n$ 。对于每对正整数 (m, n) , 下面哪个等于 12^{mn} ?

- (A) P^2Q (B) P^nQ^m (C) P^nQ^{2m} (D) $P^{2m}Q^n$ (E) $P^{2n}Q^m$

Problem 14

Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?

四个全等的矩形如图示放置。最外层大的正方形的面积是内层正方形面积的 4 倍。问每个矩形的长边的长度和短边的长度的比值是多少？

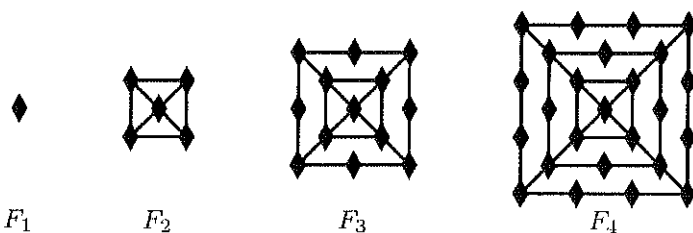


- (A) 3 (B) $\sqrt{10}$ (C) $2 + \sqrt{2}$ (D) $2\sqrt{3}$ (E) 4

Problem 15

The figures F_1 , F_2 , F_3 , and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20} ?

如下所示 F_1 , F_2 , F_3 , 和 F_4 是一系列图形中的前 4 个。对于 $n \geq 3$, F_n 是以以下方式形成：在 F_{n-1} 的外层添加一层正方形，这个正方形每条边上钻石的个数比 F_{n-1} 最外层正方形每条边上钻石的个数多 1。例如图形 F_3 有 13 个钻石。问 F_{20} 有多少个钻石？



- (A) 401 (B) 485 (C) 585 (D) 626 (E) 761

Problem 16

Let a , b , c , and d be real numbers with $|a - b| = 2$, $|b - c| = 3$, and $|c - d| = 4$.

What is the sum of all possible values of $|a - d|$?

a , b , c , 和 d 为实数, 满足 $|a - b| = 2$, $|b - c| = 3$, $|c - d| = 4$ 。问 $|a - d|$ 的所有可能值之和是多少?

- (A) 9 (B) 12 (C) 15 (D) 18 (E) 24

Problem 17

Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that EF is perpendicular to DB , and A and C lie on DE and DF , respectively. What is EF ?

矩形 $ABCD$ 中, $AB = 4$, $BC = 3$ 。线段 EF 通过点 B 满足 EF 垂直 DB , 且点 A 和 C 分别在 DE 和 DF 上。问 EF 长为多少?

- (A) 9 (B) 10 (C) $\frac{125}{12}$ (D) $\frac{103}{9}$ (E) 12

Problem 18

At Jefferson Summer Camp, 60% of the children play soccer, 30% of the children swim, and 40% of the soccer players swim. To the nearest whole percent, what percent of the non-swimmers play soccer?

在 Jefferson 夏令营, 60% 的孩子踢足球, 30% 的孩子游泳, 踢足球的孩子中有 40% 游泳。问不游泳的人当中, 百分之多少踢足球? 结果取最接近的整数百分数。

- (A) 30% (B) 40% (C) 49% (D) 51% (E) 70%

Problem 19

Circle A has radius 100. Circle B has an integer radius $r < 100$ and remains internally tangent to circle A as it rolls once around the circumference of circle A . The two circles have the same points of tangency at the beginning and end of circle B 's trip. How many possible values can r have?

圆 A 的半径为 100。圆 B 的半径 r 是个整数满足 $r < 100$ ，圆 B 内切于圆 A 且沿着圆 A 的圆周滚动了一圈。滚动前和滚动结束后这 2 个圆相切于同一点。问 r 有多少个可能值？

- (A) 4 (B) 8 (C) 9 (D) 50 (E) 90

Problem 20

Andrea and Lauren are 20 kilometers apart. They bike toward one another with Andrea traveling three times as fast as Lauren, and the distance between them decreasing at a rate of 1 kilometer per minute. After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren. After how many minutes from the time they started to bike does Lauren reach Andrea?

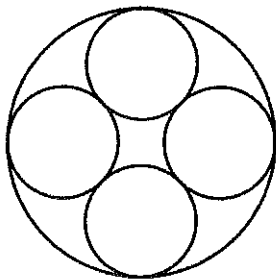
Andrea 和 Lauren 相距 20 公里。她们骑自行车相向而行，Andrea 的骑车速度是 Lauren 的 3 倍，且她们间的距离每分钟减小 1 公里。5 分钟后，Andrea 的轮胎坏了，她停止骑行并等待 Lauren。问她们出发后多少分钟 Lauren 和 Andrea 相遇？

- (A) 20 (B) 30 (C) 55 (D) 65 (E) 80

Problem 21

Many Gothic cathedrals have windows with portions containing a ring of congruent circles that are circumscribed by a larger circle. In the figure shown, the number of smaller circles is four. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger circle?

许多哥特式大教堂都有窗户，窗户的一部分包含一个由一个更大的圆包围的全等圆环。在所示的图中，较小的圆有四个。四个较小圆的面积之和与较大圆的面积之比是多少？



- (A) $3 - 2\sqrt{2}$ (B) $2 - \sqrt{2}$ (C) $4(3 - 2\sqrt{2})$ (D) $\frac{1}{2}(3 - \sqrt{2})$ (E) $2\sqrt{2} - 2$

Problem 22

Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7?

两个正方体骰子上均有数字 1 到 6 且这些数字可以移走。将这两个骰子上 12 个数字移走，放入一个包中，然后一次抽出一个，并重新随机粘贴到骰子的面上，一个面粘贴一个数字。最后再掷这 2 个骰子，并将朝上的两个面上的数字相加。

问所得的和等于 7 的概率是多少？

- (A) $\frac{1}{9}$ (B) $\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{2}{11}$ (E) $\frac{1}{5}$

Problem 23

Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E , $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE ?

凸四边形 $ABCD$ 中， $AB = 9$ 且 $CD = 12$ 。对角线 AC 和 BD 交于 E ， $AC = 14$ ，且 $\triangle AED$ 和 $\triangle BEC$ 面积相等。则 AE 是多长？

- (A) $\frac{9}{2}$ (B) $\frac{50}{11}$ (C) $\frac{21}{4}$ (D) $\frac{17}{3}$ (E) 6

Problem 24

Three distinct vertices of a cube are chosen at random. What is the probability that the plane determined by these three vertices contains points inside the cube?

从一个正方体的所有顶点中随机选择 3 个。则这 3 个点所确定的平面包含正方形内部的点的概率是多少？

- (A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{4}{7}$ (D) $\frac{5}{7}$ (E) $\frac{3}{4}$

Problem 25

For $k > 0$, let $I_k = 10 \dots 064$, where there are k zeros between the 1 and the 6.

Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k . What is the maximum value of $N(k)$?

对于 $k > 0$, 令 $I_k = 10 \dots 064$, 其中数字 1 和 6 之间有 k 个 0. 令 $N(k)$ 表示 I_k 的质因数分解中, 因子 2 的个数。则 $N(k)$ 的最大值是多少?

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10